Equipopularity in the Separable Permutations

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This talk investigates pattern occurrences within permutation classes, and classifies the patterns which occur the same number of times within the separable class.

For any permutation σ, define a permutation statistic $\nu_\sigma : \mathfrak{S} \to \mathbb{Z}_{\geq 0}$ which counts the number of occurrences of the pattern σ in a given permutation. For example, $\nu_{213}(462513) = 2$ since the first, third, and fourth entries as well as the third, fifth, and sixth entries form 213 patterns. For another example, $\nu_2 1$ counts the number of inversions of a permutation. Note that every permutation statistic can be expressed through combinations of counts of permutation patterns [4].

For a permutation class $\mathcal{C}$ and a pattern σ, define the *popularity* of σ within $\mathcal{C}$ to be the sequence

$$\nu_\sigma(\mathcal{C}_1),\ \nu_\sigma(\mathcal{C}_2),\ \nu_\sigma(\mathcal{C}_3),\ \nu_\sigma(\mathcal{C}_4),\ \ldots,$$

where $\mathcal{C}_n$ denotes the length n permutations of the class. For a given class, two patterns are *equipopular* if they have the same popularity. It is easy to show in the class of all permutations, all k-patterns are equipopular; the situation quickly becomes more interesting when restricted to proper classes.

Prior research has focused on pattern popularity within principal permutation classes. Bóna [2, 3] showed that the patterns 213, 231, and 312, were equipopular in the class Av 132 and provided full enumerations of the popularity of the length 3 patterns in this class. A similar study of the class Av 123 showed that the popularity of 231 is idential to that of 231 in Av 132 [6]. Rudolph [7] provided sufficient conditions for patterns to be equipopular in Av 132 and Chua and Sankar [5] showed that these conditions are necessary, classifying the equipopular patterns in this class.

References

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