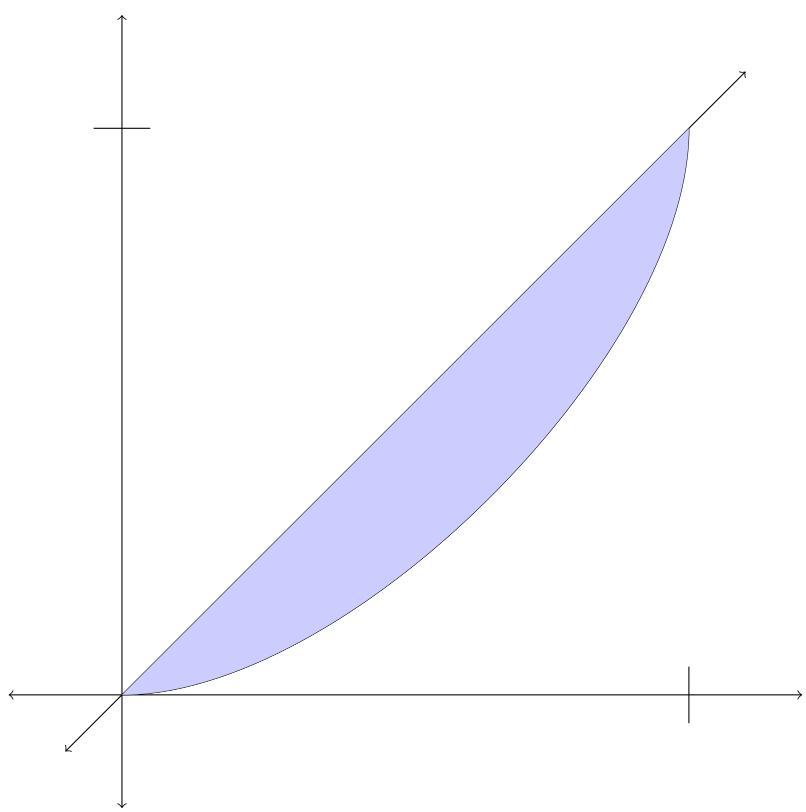
1. Let $f(x,y) = 2xy + 2x$, and R be the region of the xy-plane between the two curves $y = x$ and $y = x^2$.

   a) Sketch the region R



Answer.

□

b) Set up (but don't solve) an integral representing the volume below the surface $f(x,y)$ within the region R...

(i) by integrating first with respect to x:

Answer.

$$\int_0^1 \int_y^{\sqrt{y}} 2xy + 2x \; \mathrm{d}x\mathrm{d}y$$

□

(ii) by integrating first with respect to y:

Answer.

$$\int_0^1 \int_{x^2}^{x} 2xy + 2x\mathrm{d}y \; \mathrm{d}x$$

□

c) Find the volume by evaluating one of the integrals in part b.

Answer. For the first integral, you get:

$$\begin{aligned}
V &= \int_0^1 \int_y^{\sqrt{y}} 2xy + 2x \, \mathrm{d}x\mathrm{d}y \\
&= \int_0^1 \left(x^2y + x^2\right)\Big|_y^{\sqrt{y}} \mathrm{d}y \\
&= \int_0^1 \left(y^2 + y\right) - \left(y^3 + y^2\right) \mathrm{d}y \\
&= \int_0^1 y - y^3 \mathrm{d}y \\
&= \left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Big|_0^1 \\
&= \frac{1}{2} - \frac{1}{4} - 0 \\
&= \frac{1}{4}
\end{aligned}$$

If you chose to do the second, you should have

$$\begin{aligned}
V &= \int_0^1 \int_{x^2}^{x} 2xy + 2x \mathrm{d}y \, \mathrm{d}x \\
&= \int_0^1 \left(xy^2 + 2xy\right)\Big|_{x^2}^{x} \mathrm{d}x \\
&= \int_0^1 \left(x^3 + 2x^2\right) - \left(x^5 + 2x^3\right) \mathrm{d}x \\
&= \int_0^1 -x^5 - x^3 + 2x^2 \, \mathrm{d}x \\
&= \left(\frac{-x^6}{6} - \frac{x^4}{4} + \frac{2x^3}{3}\right)\Big|_0^1 \\
&= \frac{2}{3} - \frac{1}{4} - \frac{1}{6} \\
&= \frac{8 - 3 - 2}{12} \\
&= \frac{3}{12} = \frac{1}{4}
\end{aligned}$$

□