MAC2313 Quiz 3 Name ____________

1. (10 points) Suppose that a particle moves along the curve

$$f(t) = \left\langle \frac{2t^{3/2}}{3}, \sin t, \cos t \right\rangle, \qquad 0 \le t \le 3.$$

(assume that t is in seconds, and the coordinates are in meters)

a) What is the speed of the particle at time $t = 0$? at $t = 3$?

Answer. Speed is the *magnitude* of the tangent vector. The tangent vector is given by

$$f'(t) = \left\langle t^{1/2}, \cos t, -\sin t \right\rangle.$$

At $t = 0$, this becomes $f'(0) = \langle 0, 1, 0 \rangle$, which has magnitude $\sqrt{0^2 + 1^2 + 0^2} = 1$.
At $t = 3$, this is $f'(3) = \left\langle \sqrt{3}, \cos 3, -\sin 3 \right\rangle$, which has magnitude $\sqrt{\sqrt{3}^2 + \cos^2 3 + \sin^2 3} = \sqrt{3+1} = 2$. □

Answer: 1 meter per second

Answer: 2 meters per second

b) How far does the particle travel (arc length)?

Answer. The arc length is given by

$$\begin{aligned}
\int_0^3 |f'(t)|\,\mathrm{d}t &= \int_0^3 \left|\left\langle t^{1/2}, \cos t, -\sin t\right\rangle\right|\,\mathrm{d}t \\
&= \int_0^3 \sqrt{(t^{1/2})^2 + \cos^2 t + \sin t^2}\,\mathrm{d}t \\
&= \int_0^3 \sqrt{t+1}\,\mathrm{d}t \\
&= \int_{t=0}^{t=3} \sqrt{u}\,\mathrm{d}u \qquad u = t+1,\ \mathrm{d}u = \mathrm{d}t \\
&= \left(\frac{2u^{3/2}}{3}\right)\Bigg|_{t=0}^{t=3} \\
&= \left(\frac{2(t+1)^{3/2}}{3}\right)\Bigg|_{t=0}^{t=3} \\
&= \frac{2(4)^{3/2}}{3} - \frac{2(1)^{3/2}}{3} \\
&= \frac{2\cdot 8}{3} - \frac{2}{3} \\
&= \frac{14}{3}.
\end{aligned}$$

□

Answer: 14/3 meters

c) What is the *average* speed of the particle?

Answer. If you travel 14/3 meters in three seconds, you average $\frac{14/3}{3} = \frac{14}{9}$ meters every second. □

Answer: 14/9 meters per second