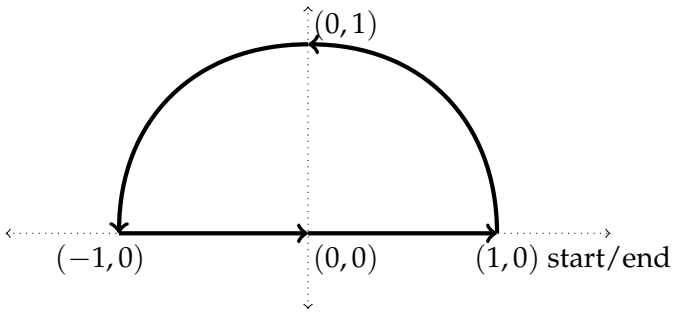
1. The water current at each point (x, y) in a lake is given by $F(x, y) = \langle 1 - y, x \rangle$. You row your boat from the point $(1, 0)$, along the top half of a semi-circle to the point $(-1, 0)$, and then back along a straight line, as shown below:



a) How much work is done by the current...

(i) (4 points)... during the first part of the path (the semicircle)?

Answer. The parameterization of the circle is given by $r(t) = \langle \cos(t), \sin(t) \rangle$ with $0 \leq t \leq \pi$, and so $r'(t) = \langle -\sin(t), \cos(t) \rangle$, and by substituting for x, y we get that the vector field along this curve is $F(t) = \langle 1 - \sin(t), \cos(t) \rangle$. Therefore, the work is given by

$$\begin{aligned}
\int_C \mathbf{F} \cdot \mathbf{T} ds &= \int_0^\pi F \cdot r'(t) \, dt \\
&= \int_0^\pi \langle 1 - \sin(t), \cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle \, dt \\
&= \int_0^\pi (-\sin(t) + \sin^2(t)) + (\cos^2(t)) \, dt \\
&= \int_0^\pi -\sin(t) + 1 \, dt \\
&= \int_0^\pi -\sin(t) \, dt + \int_0^1 1 \, dt \\
&= (\cos(t))\Big|_0^\pi + (t)\Big|_0^\pi \\
&= (-1 - 1) + \pi \\
&= \pi - 2.
\end{aligned}$$

□

(ii) (4 points)...during the second part of the path (the line)?

Answer. The parameterization of the straight line between $(-1,0)$ and $(1,0)$ is given by $(1-t)\langle -1,0\rangle + t\langle 1,0\rangle = \langle t-1+t,0\rangle = \langle 2t-1,0\rangle$, and so $r(t) = \langle 2t-1,0\rangle$ for $0 \leq t \leq 1$ and $r'(t) = \langle 2,0\rangle$. Substituting t for x, y gives $F(t) = \langle 1, 2t-1\rangle$. Therefore, the work done by the current is given by

$$\begin{aligned}\int_C \mathbf{F}\cdot\mathbf{T}\mathrm{d}s &= \int_0^1 F\cdot r'(t)\,\mathrm{d}t \\ &= \int_0^1 \langle 1,2t-1\rangle\cdot\langle 2,0\rangle\,\mathrm{d}t \\ &= \int_0^1 2+0\,\mathrm{d}t \\ &= (2t)\Big|_0^1 \\ &= 2.\end{aligned}$$

□

b) (2 points) Is the vector field $F(x,y)$ conservative? why or why not?

Answer. No. If it was conservative, then the work done over a closed loop would be zero. However, adding up the two integrals above does *not* give zero.
Alternatively, you could use the second derivative test to see that the field is not conservative. □

c) (0 points) What was your favorite part of this class?

Answer. ALL OF IT □