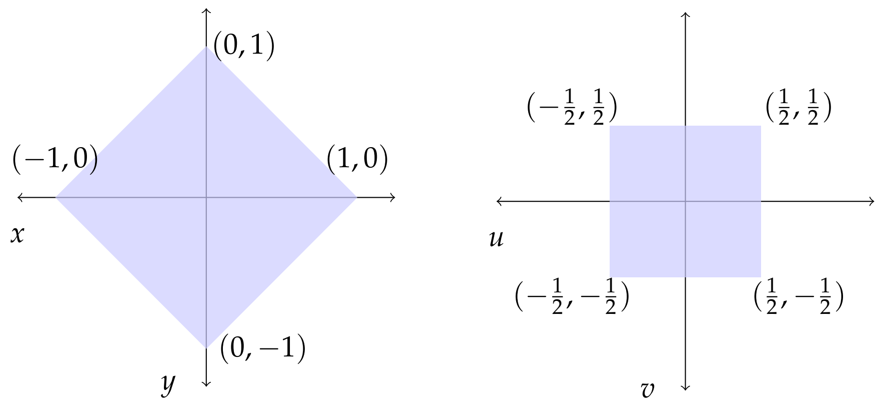
1. Let R be the region in the xy plan bounded by the lines $y = x+1$, $y = x-1$, $y = -x+1$, $y = -x-1$, and let u, v be defined by $x = u - v$, $y = u + v$.

   a) Sketch the region R in both the xy plane and the uv plane:



   b) Find the Jacobian of the transformation $(x, y) \mapsto (u, v)$.

$$J = \det \begin{pmatrix} \partial_u x & \partial_v x \\ \partial_u y & \partial_v y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = 1 + 1 = 2.$$

   c) Let $f(x, y) = (x + y)^2$. Rewrite the integral

$$\int\int_R f(x, y) \mathrm{d}x \mathrm{d}y$$

   in terms of u and v, and solve the integral.

$$\begin{aligned} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} (2u)^2 |J| \ \mathrm{d}v \ \mathrm{d}u &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{2}}^{\frac{1}{2}} 4u^2 2 \ \mathrm{d}v \ \mathrm{d}u \\ &= 8 \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(u^2 v \right) \Big|_{-\frac{1}{2}}^{\frac{1}{2}} \ \mathrm{d}u \\ &= 8 \int_{-\frac{1}{2}}^{\frac{1}{2}} u^2 \ \mathrm{d}u \\ &= 8 \left(\frac{u^3}{3} \right) \Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} \\ &= 8 \left(\frac{1}{24} - \frac{-1}{24} \right) = \frac{2}{3}. \end{aligned}$$