Homework Problems

Integration by Parts

Directions: Evaluate.

1. $\displaystyle\int \ln(2x+3)\,dx$

2. $\displaystyle\int 5x\sec^2 9x\,dx$

3. $\displaystyle\int_1^{\sqrt{3}} 55\arctan(1/x)\,dx$

4. $\displaystyle\int_1^2 \frac{9(\ln x)^2}{x^3}\,dx$

5. $\displaystyle\int_0^6 \frac{x^3\,dx}{\sqrt{4+x^2}}$

6. $\displaystyle\int 13\cos\sqrt{x}\,dx$

7. $\displaystyle\int x^3 e^{-x^2}\,dx$

8. $\displaystyle\int_0^{\pi} e^{\cos x}\sin 2x\,dx$

9. $\displaystyle\int 25\sin(\ln x)\,dx$

Integrating Trigonometric Functions

Directions: Evaluate.

10. $\displaystyle\int 15\sin^3 x\cos^2 x\,dx$

11. $\displaystyle\int \frac{12\sin^3\sqrt{x}}{\sqrt{x}}\,dx$

12. $\displaystyle\int_0^{\pi/2} 23\sin^2 2x\,dx$

13. $\displaystyle\int_0^{\pi} 69\sin^2 x\cos^4 x\,dx$

14. $\displaystyle\int \frac{\cos^5 x}{\sqrt{\sin x}}\,dx$

15. $\displaystyle\int_0^{\pi/2} 27\sec^4(x/2)\,dx$

16. $\displaystyle\int 18(\tan^2 x+\tan^4 x)\,dx$

17. $\displaystyle\int_0^{\pi/4} 12\sec^4 x\tan^4 x\,dx$

18. $\displaystyle\int 47\tan^3 x\sec x\,dx$

19. $\displaystyle\int 59\tan^2 x\sec x\,dx$

20. $\displaystyle\int 43\cos(\pi x)\cos(4\pi x)\,dx$

21. $\displaystyle\int \frac{69\,dx}{\cos x-1}$

Trigonometric Substitution

Directions: Evaluate.

22. $\displaystyle\int_{3\sqrt{2}}^6 \frac{dx}{x^3\sqrt{x^2-9}}$

23. $\displaystyle\int \frac{x^5\,dx}{\sqrt{x^2+7}}$

24. $\displaystyle\int \sqrt{1-81x^2}\,dx$

25. $\displaystyle\int \sqrt{40+6x-x^2}\,dx$

26. $\displaystyle\int \frac{9x\,dx}{\sqrt{x^2+x+1}}$

27. $\displaystyle\int \frac{x^2\,dx}{(15+6x-9x^2)^{3/2}}$

28. $\displaystyle\int \sqrt{x^2+18x}\,dx$

29. $\displaystyle\int 7x\sqrt{1-x^4}\,dx$

30. $\displaystyle\int_{-\pi/2}^0 \frac{\cos x\,dx}{\sqrt{1+\sin^2 x}}$

31. $\displaystyle\int \frac{x^2\,dx}{(9-x^2)^{3/2}}$

Integrating Rational Functions

Directions: Evaluate.

32. $\displaystyle\int_0^1 \frac{x^3-x-8}{x^2-x-6}\,dx$

33. $\displaystyle\int \frac{2x^2-11x+19}{(2x+1)(x-2)^2}\,dx$

34. $\displaystyle\int \frac{x^3+1}{x^2+9}\,dx$

35. $\displaystyle\int \frac{34\,dx}{(x-5)(x^2+9)}$

36. $\displaystyle\int \frac{x^2+5x+1}{(x^2+1)^2}\,dx$

37. $\displaystyle\int \frac{7\,dx}{x^3-1}$

38. $\displaystyle\int_1^3 \frac{x\,dx}{x^2+4x+13}$

39. $\displaystyle\int \frac{3x^3+18x^2+33x-2}{(x^2+6x+10)^2}\,dx$

40. $\displaystyle\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$

41. $\displaystyle\int \frac{\cos x\,dx}{7\sin^2 x+2\sin x}$

42. $\displaystyle\int \frac{e^x\,dx}{(e^x-9)(e^{2x}+16)}$

43. $\displaystyle\int 11\ln(x^2-x+6)\,dx$

44. $\displaystyle\int \frac{dx}{x^2-4}$

45. $\displaystyle\int \frac{x+1}{x^2+5x+4}\,dx$

46. $\displaystyle\int \frac{x^2+x+7}{(x^2+7)^2}\,dx$

47. $\displaystyle\int \frac{8x^3\,dx}{x^2+1}$

48. $\displaystyle\int_1^3 \frac{6x-1}{x^2(x+1)}\,dx$

49. $\displaystyle\int_7^8 \frac{x\,dx}{x^2-4x-5}$

50. $\displaystyle\int \frac{6x-4}{3x^2-4x+7}\,dx$

Integration Strategies

Directions: Evaluate.

51. $\displaystyle\int 51\tan^3\theta\,d\theta$

52. $\displaystyle\int_{-1}^{1/\sqrt{3}} \frac{5e^{\arctan y}}{1+y^2}\,dy$

53. $\displaystyle\int \frac{9x\,dx}{x^4+x^2+4}$

54. $\displaystyle\int_0^1 \left(1+\sqrt{x}\right)^8\,dx$

55. $\displaystyle\int 6\ln(x^2-25)\,dx$

56. $\displaystyle\int \frac{18\,dx}{e^x+18}$

57. $\displaystyle\int \theta\tan^2\theta\,d\theta$

58. $\displaystyle\int 5\sqrt{9+e^x}\,dx$

59. $\displaystyle\int 4x^5e^{-x^3}\,dx$

60. $\displaystyle\int \frac{dx}{x\sqrt{6x+49}}$

61. $\displaystyle\int \frac{dx}{x\sqrt{4x^2+1}}$

62. $\displaystyle\int \frac{dx}{e^{3x}-e^x}$

63. $\displaystyle\int \frac{dx}{(x-7)(x^2+16)}$

64. $\displaystyle\int \frac{7x\ln x}{\sqrt{x^2-16}}\,dx$

65. $\displaystyle\int x\sqrt[7]{x+a}\,dx$

66. $\displaystyle\int \frac{12\,dx}{\sqrt{x}+x\sqrt{x}}$

Improper Integrals

Directions: Evaluate.

67. $\displaystyle\int_1^\infty \frac{4\,dx}{(5x+1)^2}$

68. $\displaystyle\int_0^\infty \frac{x\,dx}{(x^2+32)^2}$

69. $\displaystyle\int_4^\infty 35e^{-y/2}\,dy$

70. $\displaystyle\int_0^\infty \frac{5\,dz}{z^2+7z+6}$

71. $\displaystyle\int_1^\infty \frac{67\ln x}{x}\,dx$

72. $\displaystyle\int_0^\infty \frac{7x\arctan x}{(1+x^2)^2}\,dx$

Limits and L'Hôpital's Rule

Directions: Evaluate.

73. $\displaystyle\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$

74. $\displaystyle\lim_{x\to 0} x^2\sin(1/x)$

75. $\displaystyle\lim_{u\to\infty}\frac{3u^4+2}{(u^2-4)(3u^2-1)}$

76. $\displaystyle\lim_{x\to\infty}\left(\sqrt{81x^2+x}-9x\right)$

77. $\displaystyle\lim_{x\to\infty} e^{-2x}\cos x$

78. $\displaystyle\lim_{x\to 0}(1+x)^{1/x}$

Infinite Sequences

Directions: Determine whether the sequence converges or diverges. If it converges, find the limit.

79. $a_n=\dfrac{2^{n+6}}{7^n}$

80. $a_n=\tan\left(\dfrac{3n\pi}{4+12n}\right)$

81. $a_n=\dfrac{(-1)^{n+4}n^3}{n^3+4n^2+4}$

82. $a_n=\dfrac{(2n-1)!}{(2n+1)!}$

83. $a_n=\dfrac{\ln(3n)}{\ln(9n)}$

84. $a_n=n^2e^{-5n}$

85. $a_n=5n\cos(5n\pi)$

86. $a_n=\dfrac{\cos^6 n}{4^n}$

87. $a_n=\dfrac{n}{5}\sin(5/n)$

88. $a_n=\left(1+\dfrac{6}{n}\right)^{2n}$

89. $a_n=\ln(3n^2+5)-\ln(n^2+5)$

90. $a_n=\dfrac{7n!}{3^n}$

91. $a_n=\dfrac{2n!}{n^{2n}}$

92. $\left\{\sqrt{2},\sqrt{2\sqrt{2}},\sqrt{2\sqrt{2\sqrt{2}}},\ldots\right\}$

Directions: Determine (a) whether the sequence is increasing, decreasing, or not monotonic; and (b) whether or not the sequence is bounded.

93. $a_n=(-3)^{n+5}$

94. $a_n=\dfrac{1}{5+2n}$

Infinite Series

Directions: Find the sum of the series.

95. $\sum_{n=1}^{\infty} \tan(3n)$

96. $\sum_{n=2}^{\infty} \frac{4}{n(n+2)}$

97. $\sum_{n=1}^{\infty} 4(0.3)^{n-1}$

Directions: Determine (a) whether $\{a_n\}$ is convergent, and (b) whether $\sum_{n=1}^{\infty} a_n$ is convergent.

98. $a_n = \frac{7n}{2n+1}$

Directions: Determine whether the series converges or diverges. If it is convergent, find the sum.

99. $\sum_{n=1}^{\infty} \frac{1+5^n}{8^n}$

100. $\sum_{n=1}^{\infty} \sqrt[n]{20}$

101. $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{8n^2+5}\right)$

102. $\sum_{k=1}^{\infty} \cos^k 1$

103. $\sum_{n=1}^{\infty} \left(\frac{3}{e^n} + \frac{2}{n(n+1)}\right)$

104. $\sum_{n=1}^{\infty} \frac{e^n}{n^7}$

105. $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

106. $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$

Directions: Determine (a) the values of x for which the series converges, and (b) the sum of the series for those values.

107. $\sum_{n=1}^{\infty} \frac{x^n}{4^n}$

Directions: Answer the questions.

108. If $\sum a_n$ and $\sum b_n$ are both divergent, is $\sum(a_n + b_n)$ necessarily divergent?

109. Consider the series $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$.

(a) Find the partial sums s_1, s_2, s_3, and s_4. Do you recognize the denominators?

(b) Use the pattern to get a formula for s_n.

(c) Show that the given infinite series is convergent, and find its sum.

Integral Test

Directions: Determine whether the series converges or diverges.

110. $\sum_{n=1}^{\infty} ne^{-6n}$

111. $\sum_{n=1}^{\infty} \frac{3\ln n}{n^5}$

112. $\sum_{n=1}^{\infty} \frac{e^{1/n^9}}{n^{10}}$

113. $\sum_{n=1}^{\infty} \frac{n^2}{e^{9n}}$

114. $\sum_{n=1}^{\infty} \frac{n^9}{n^{20}+1}$

Directions: Find the values of p for which the series is convergent.

115. $\sum_{n=2}^{\infty} \frac{5}{n(\ln n)^p}$

Comparison Tests

Directions: Determine whether the series converges or diverges.

116. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^2\sqrt{n}}$

117. $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{4+11^n}$

118. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2+4}$

119. $\displaystyle\sum_{n=0}^{\infty} \frac{3+\cos n}{7^n}$

120. $\displaystyle\sum_{n=5}^{\infty} \frac{6\sqrt{n}}{n-4}$

121. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5^n}{n+8^n}$

122. $\displaystyle\sum_{n=1}^{\infty} \frac{2+n+2n^2}{\sqrt{1+n^2+n^6}}$

123. $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-4n}$

124. $\displaystyle\sum_{n=1}^{\infty} 5\sin(3/n)$

Alternating Series Test

Directions: Determine whether the series converges or diverges.

125. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(4n+5)}$

126. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+3\sqrt{n}}$

127. $\displaystyle\sum_{n=3}^{\infty} (-1)^{n-1} \frac{4e^{1/n}}{n}$

128. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{10n}{\ln n}$

129. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$

130. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{3/5}}$

131. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin(\pi/n)$

Directions: Show that the series is convergent. According to the Alternating Series Sum Estimation Theorem, how many terms of the series do we need to add in order to find the sum to the indicated accuracy?

132. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^7}$ $(|\text{error}| < 0.00005)$

133. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$ $(|\text{error}| < 0.0001)$

134. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{8^n n!}$ $(|\text{error}| < 0.000005)$

Directions: Approximate the sum of the series correct to four decimal places.

135. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1} n^2}{8^n}$

Ratio and Root Tests

Directions: Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

136. $\displaystyle\sum_{n=0}^{\infty} \frac{(-11)^n}{n!}$

137. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{n^4}$

138. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+5}}$

139. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{e^{1/n}}{n^7}$

140. $\displaystyle\sum_{n=1}^{\infty}\frac{\sin 4n}{7^n}$

141. $\displaystyle\sum_{n=1}^{\infty}\frac{12^n}{(n+1)8^{2n+1}}$

142. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^8 5^n}{n!}$

143. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\arctan n}{n^{12}}$

144. $\displaystyle\sum_{n=1}^{\infty}\frac{10-\sin 3n}{n^{2/3}-2}$

145. $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln(2n)}$

146. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n^2+5}{3n^2+2}\right)^n$

147. $\displaystyle\sum_{n=2}^{\infty}\left(\frac{-2n}{n+1}\right)^{2n}$

148. $\displaystyle\sum_{n=2}^{\infty}9\left(1+\frac{1}{n}\right)^{n^2}$

149. $\displaystyle 1-\frac{1\cdot 3}{3!}+\frac{1\cdot 3\cdot 5}{5!}-\frac{1\cdot 3\cdot 5\cdot 7}{7!}+\cdots+(-1)^{n+3}\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n+3)}{(2n+3)!}+\cdots$

150. $\displaystyle\sum_{n=1}^{\infty}\frac{9\cdot 18\cdot 27\cdot\cdots\cdot(9n)}{n!}$

151. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n n!}{5\cdot 8\cdot 11\cdot\cdots\cdot(3n+2)}$

152. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos n}{4n^2}$

153. $\displaystyle\sum_{n=1}^{\infty}\frac{n^n}{n!}$

Directions: For each of the following series, is the Ratio Test conclusive or inconclusive?

154. (a) $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n^3}$

(b) $\displaystyle\sum_{n=3}^{\infty}\frac{n}{3^n}$

(c) $\displaystyle\sum_{n=2}^{\infty}\frac{(-4)^{n-1}}{\sqrt{n}}$

(d) $\displaystyle\sum_{n=2}^{\infty}\frac{4\sqrt{n}}{1+n^2}$

Strategy for Testing Series

Directions: Test the series for convergence or divergence.

155. $\displaystyle\sum_{n=5}^{\infty}\frac{1}{n\sqrt{\ln(9n)}}$

156. $\displaystyle\sum_{n=0}^{\infty}\frac{n!}{6\cdot 13\cdot 20\cdot\cdots\cdot(7n+6)}$

157. $\displaystyle\sum_{n=1}^{\infty}(-1)^n 9^{1/n}$

158. $\displaystyle\sum_{n=1}^{\infty}\tan(3/n)$

159. $\displaystyle\sum_{k=1}^{\infty}\frac{12^k}{3^k+5^k}$

160. $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^n}{n^{6n}}$

161. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+n\cos^2 3n}$

162. $\displaystyle\sum_{n=1}^{\infty}n^2 e^{-4n^3}$

163. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^3+2}}{6n^3+4n^2+5}$

164. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^7}$

165. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{2n}}{n^n}$

Power Series

Directions: Find the radius of convergence and interval of convergence of the series.

166. $\displaystyle\sum_{n=3}^{\infty} \frac{x^{n+11}}{\sqrt{n}}$

167. $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n x^{n+8}}{n+5}$

168. $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n-1} x^{n+4}}{n^3}$

169. $\displaystyle\sum_{n=3}^{\infty} \sqrt{n}\, x^{n+5}$

170. $\displaystyle\sum_{n=1}^{\infty} \frac{x^{n+8}}{8n!}$

171. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{11^n \ln n}$

172. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-10)^n}{n^2+1}$

173. $\displaystyle\sum_{n=2}^{\infty} \frac{x^{8n}}{n(\ln n)^4}$

174. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\, x^n}{6 \cdot 13 \cdot 20 \cdot \cdots \cdot (7n-1)}$

Directions: Suppose that $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?

175. (a) $\displaystyle\sum_{n=0}^{\infty} c_n$

(b) $\displaystyle\sum_{n=0}^{\infty} c_n 9^n$

(c) $\displaystyle\sum_{n=0}^{\infty} c_n (-3)^n$

(d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

Representations of Functions as Power Series

Directions: Find a power series representation for the function and determine the interval of convergence.

176. $f(x) = \dfrac{1}{6+x}$

177. $f(x) = \dfrac{9}{4-x}$

178. $f(x) = \dfrac{x}{49+x^2}$

179. $f(x) = \dfrac{x}{5x^2+1}$

Directions: Find a power series representation for the function and determine the radius of convergence.

180. (a) $f(x) = \dfrac{1}{(4+x)^2}$

(b) $f(x) = \dfrac{1}{(4+x)^3}$

(c) $f(x) = \dfrac{x^2}{(4+x)^3}$

181. (a) $f(x) = \ln(1+x)$

(b) $f(x) = x\ln(1+x)$

(c) $f(x) = \ln(x^2+1)$

182. $f(x) = \ln(2-x)$

183. $f(x) = \dfrac{x^3}{(x-9)^2}$

184. $f(x) = \arctan(x/8)$

Directions: Evaluate the indefinite integral as a power series and determine the radius of convergence.

185. $\displaystyle\int \frac{\ln(1-t)}{t}\,dt$

186. $\displaystyle\int \frac{x-\arctan x}{x^3}\,dx$

187. $\displaystyle\int \arctan(x^2)\,dx$

Directions: Use a power series to approximate the definite integral to six decimal places.

188. $\displaystyle\int_0^{0.5} \ln(1+x^6)\,dx$

189. $\displaystyle\int_0^{0.3} \frac{dx}{1+x^6}$

Directions: Use the formula

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

to compute the indicated value correct to five decimal places.

190. $\ln(1.09)$

Taylor and Maclaurin Series

Directions: Answer the questions.

191. Find the Taylor series for f centered at 2 and the radius of convergence given that $f^{(n)}(2) = \dfrac{(-1)^n n!}{4^n(n+3)}$.

192. Find the Maclaurin series for f and the radius of convergence given that $f(x) = \ln(1+5x)$.

193. Find the Maclaurin series for f and the radius of convergence given that $f(x) = \sin(\pi x/3)$.

194. Find the Maclaurin series for f and the radius of convergence given that $f(x) = xe^x$.

195. Find the Taylor series for f centered at 2 given that $f(x) = x^4 - 6x^2 + 1$.

196. Find the Taylor series for f centered at -4 given that $f(x) = 6/x$.

197. Find the Taylor series for f centered at 9π given that $f(x) = 5\cos x$.

198. Find the Taylor series for f centered at 16 given that $f(x) = 1/\sqrt{x}$.

199. Use the Maclaurin series for e^x to compute $e^{-0.18}$ correct to five decimal places.

200. Use the Maclaurin series for $\sin x$ to compute $2\sin(1^\circ)$ correct to five decimal places.

201. Evaluate the indefinite integral as an infinite series: $\displaystyle 9\int \frac{e^x - 1}{13x}\,dx$.

202. Evaluate the indefinite integral as an infinite series: $\displaystyle\int \arctan(x^2)\,dx$.

203. Use series to approximate the definite integral correct to three decimal places: $\displaystyle\int_0^1 x\cos(x^5)\,dx$.

204. Use series to evaluate the limit: $\displaystyle\lim_{x\to 0} \frac{5x - \arctan(5x)}{x^3}$.

Curves Defined by Parametric Equations

Directions: Eliminate the parameter to find a Cartesian equation of the curve.

205. $x = e^t - 6$, $y = e^{2t}$

206. $x = \ln t$, $y = \sqrt{t}$, $t \geq 36$

Directions: Determine what curve is represented by the parametric equations. Be sure to indicate direction as well as any starting or ending points.

207. $x = 2\cos(3t)$, $y = 2\sin(3t)$, $0 \le t \le 2\pi/3$

208. $x = \sin(4t)$, $y = \sin^2(4t)$

Calculus with Parametric Curves

Directions: Answer the questions.

209. Find an equation of the tangent to $x = t^4 + 2$, $y = t^3 + t$ at the point corresponding to $t = 1$.

210. Find an equation of the tangent to $x = 2 + \ln t$, $y = t^2 + 2$ at the point $(2, 3)$ by both eliminating the parameter and without eliminating the parameter.

211. Consider $x = 3 + t^2$, $y = t^2 + t^3$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

(b) For which values of t is the curve concave upward?

212. Consider $x = 10 - t^2$, $y = t^3 - 27t$.

(a) Find the points on the curve where the tangent is horizontal.

(b) Find the points on the curve where the tangent is vertical.

213. Consider $x = 3\cos\theta$, $y = \sin 2\theta$.

(a) Find the points on the curve where the tangent is horizontal.

(b) Find the points on the curve where the tangent is vertical.

214. Consider $x = 5\cos t$, $y = 2\sin t\cos t$. Show that this curve has two tangents at $(0, 0)$ and find their equations.

215. Consider $x = t - t^2$, $y = 4t^{3/2}/3$, $3 \le t \le 5$. Setup an integral that represents the length of the curve and then evaluate that integral.

216. Find the length of the curve: $x = 1 + 3t^2$, $y = 9 + 2t^3$, $0 \le t \le 3$.

217. Find the length of the curve: $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 3$.

Polar Coordinates

Directions: Answer the questions.

218. The Cartesian coordinates of a point are given.

(a) Find polar coordinates (r, θ) of the point $(2, -2)$, where $r > 0$ and $0 \le \theta \le 2\pi$.

(b) Find polar coordinates (r, θ) of the point $(2, -2)$, where $r < 0$ and $0 \le \theta \le 2\pi$.

(c) Find polar coordinates (r, θ) of the point $(1, \sqrt{3})$, where $r > 0$ and $0 \le \theta \le 2\pi$.

(d) Find polar coordinates (r, θ) of the point $(1, \sqrt{3})$, where $r < 0$ and $0 \le \theta \le 2\pi$.

219. Find a Cartesian equation for the polar curve $r = 6\sin\theta$ and identify the curve.

Directions: Sketch the graph of the given polar equation.

220. $r = \sin\theta$

221. $r = -7\cos\theta$

222. $r = 5(1 - \sin\theta)$, $\theta \ge 0$

223. $r = 2 - 4\cos\theta$

224. $r = \theta$, $\theta \ge 0$

225. $r = 2\sin(6\theta)$

226. $r = \cos(7\theta)$

227. $r = 6\cos(2\theta)$

228. $r = 4\cos(6\theta)$

229. $r = 2 - 3\sin\theta$

230. $r = 3 + 2\sin\theta$

231. $r^2 = 6\sin(2\theta)$

Directions: Answer the questions.

232. Find the slope of the tangent line to the polar curve $r = 7\sin\theta$ at the point specified by $\theta = \pi/3$.

233. Consider the polar equation $r = 3\cos\theta$, $0 \le \theta < \pi$.

(a) Find the θ-values of the points on the curve where the tangent line is horizontal.

(b) Find the θ-values of the points on the curve where the tangent line is vertical.

234. Consider the polar equation $r = 1 - \cos\theta$, $0 \le \theta < 2\pi$.

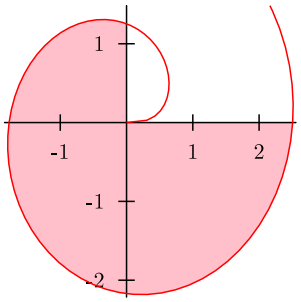
(a) Find the θ-values of the points on the curve where the tangent line is horizontal.

(b) Find the θ-values of the points on the curve where the tangent line is vertical.

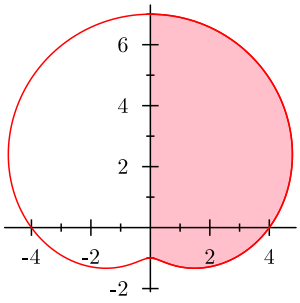
Areas in Polar Coordinates

Directions: Answer the questions.

235. Find the area of the shaded region for $r = \sqrt{\theta}$.



236. Find the area of the shaded region for $r = 4 + 3\sin\theta$.



237. Find the area enclosed by $r = 13(1 + \cos\theta)$.

238. Find the area enclosed by $r^2 = 11\cos(2\theta)$.

239. Find the area enclosed by $r = 3 - \sin\theta$.

240. Find the area enclosed by $r = 6\cos(3\theta)$.

241. Find the area of the region enclosed by one loop of the curve $r = \sin(4\theta)$.

242. Find the area of the region enclosed by the inner loop of the curve $r = 3 + 6\sin\theta$.

243. Find the area of the region that lies inside the first curve and outside the second curve:

$$r = 4\cos\theta,$$
$$r = 2.$$

244. Find the area of the region that lies inside the first curve and outside the second curve:

$$r = 6 - 6\sin\theta,$$
$$r = 6.$$

245. Find the area of the region that lies inside the first curve and outside the second curve:

$$r^2 = 50\cos(2\theta),$$
$$r = 5.$$

246. Find the area of the region that lies inside the first curve and outside the second curve:

$$r = 9\cos\theta,$$
$$r = 4 + \cos\theta.$$

247. Find the area of the region that lies inside both curves:

$$r = \cos\theta,$$
$$r = \sin\theta.$$

248. Find the area of the region that lies inside both curves:

$$r^2 = \sin(2\theta),$$
$$r^2 = \cos(2\theta).$$

249. Find the area inside the larger loop and outside the smaller loop of the limaçon $r = \dfrac{\sqrt{3}}{2} + \cos\theta$.

250. Find the polar coordinates with $0 \le \theta < 2\pi$ of all points of intersection of the curves:

$$r = 1 + \sin\theta,$$
$$r = 3\sin\theta.$$

251. Find the polar coordinates with $0 \le \theta < 2\pi$ of all points of intersection of the curves:

$$r = 10\sin(2\theta),$$
$$r = 5.$$

252. Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

Areas and Volumes

Directions: Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

253. $y = 7\sin x$, $y = 0$, $\pi/2 \le x \le \pi$; about the x-axis

254. $y = 3\sin x$, $y = 3\cos x$, $0 \le x \le \pi/4$; about $y = 3$

255. $x = 4\sqrt{y}$, $x = 0$, $y = 1$; about the y-axis

256. $y = 5x^4$, $y = 5x$, $x \ge 0$; about the x-axis

257. $y = x$, $y = \sqrt{x}$; about $y = 1$

Directions: Answer the questions.

258. Find the volume of a frustum of a right circular cone with height h, lower base radius R, and top radius r.

259. Consider a frustum of a pyramid with square base of side b, square top of side a, and height h.

(a) Find the volume if $b = 7$, $a = 5$, and $h = 4$.

(b) What are the shape and volume if $a = b$? (Express your answer in terms of b and h.)

(c) What are the shape and volume if $a = 0$? (Express your answer in terms of b and h.)

260. Find the area of the region enclosed by the curves: $y = 3\sin x$, $y = e^{4x}$, $x = 0$, $x = \pi/2$.

Volumes by Cylindrical Shells

Directions: Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

261. $y = e^x$, $y = e^{-x}$, $x = 1$; about the y-axis

262. $y = e^x$, $x = 0$, $y = 4\pi$; about the x-axis

263. $y = 10x(x-2)^2$, $y = 0$; about the y-axis

264. $y = 5/x$, $y = 0$, $x = 1$, $x = 6$; about the y-axis

265. $y = 2 + x - x^2$, $x + y = 2$; about the y-axis

266. $y = x^4$, $y = 0$, $x = 2$; about $x = 3$

267. $y = 7x - x^2$, $y = 10$; about $x = 2$

268. $y = x^2$, $x = y^2$; about $y = -3$

Work

269. A 400 lb cable is 200 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

270. A force of 45 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 25 cm. How much work is done in stretching the spring from 25 cm to 27 cm?

271. (a) How much work is done in lifting a 1.1 kg book off the floor to put it on a desk that is 0.7 m high? (Use the fact that the acceleration due to gravity is 9.8 m/s^2.)

(b) How much work is done in lifting a 24 lb weight 5 ft off the ground?

272. A spherical tank of radius 10.5 m is half full of oil that has a density of 900 kg/m^3. Find the work required to pump the oil out of a vertical spout of length 3.50 m set on top of the tank. (Use 9.80 for g and 3.14 for π, and round your answer to three significant digits.)

273. A tank in the shape of an inverted frustum of a right circular cone has upper radius 10 ft, lower radius 4 ft, and height 12 ft, and it is full of water. Find the work required to pump the water out of a spout that is level with the top of the tank. (Use 62.5 lb/ft^3 for the weight of water and 3.14 for π, and round your answer to three significant digits.)

274. An 8 m long trough, with vertical cross-sections in the shape of an inverted isosceles triangle with base length 3 m and height 3 m, is full of water. Find the work required to pump the water out of a vertical spout of length 2 m set on top of the tank. (Use 9.8 for g.)

275. A heavy rope, 50 ft long, weighs 0.6 lb/ft and hangs over the edge of a building 120 ft high.

(a) How much work is done in pulling the rope to the top of the building?

(b) How much work is done in pulling half the rope to the top of the building?

276. If 10 J of work is needed to stretch a spring from 12 cm to 14 cm and 14 J is needed to stretch it from 14 cm to 16 cm, then what is the natural length of the spring?

277. A force of 15 lb is required to hold a spring stretched 10 in. beyond its natural length. How much work is done in stretching it from its natural length to 16 in. beyond its natural length?