Homework Problem Answers

Integration by Parts

Directions: Evaluate.

1. $\frac{1}{2}(2x+3)\ln(2x+3) - x + C$
2. $\frac{5}{9}x\tan 9x - \frac{5}{81}\ln|\sec 9x| + C$
3. $\frac{55}{12}\left(2\pi\sqrt{3} - 3\pi + 6\ln 2\right)$
4. $\frac{27}{16} - \frac{9}{8}\ln 2 - \frac{9}{8}(\ln 2)^2$
5. $(16 + 56\sqrt{10})/3$
6. $26\sqrt{x}\sin\sqrt{x} + 26\cos\sqrt{x} + C$
7. $-\frac{1}{2}(1+x^2)e^{-x^2} + C$
8. $4/e$
9. $\frac{25}{2}x\left[\sin(\ln x) - \cos(\ln x)\right] + C$

Integrating Trigonometric Functions

Directions: Evaluate.

10. $3\cos^5 x - 5\cos^3 x + C$
11. $8\cos^3\sqrt{x} - 24\cos\sqrt{x} + C$
12. $23\pi/4$
13. $69\pi/16$
14. $2\sin^{1/2} x - \frac{4}{5}\sin^{5/2} x + \frac{2}{9}\sin^{9/2} x + C$
15. 72
16. $6\tan^3 x + C$
17. $144/35$
18. $\frac{47}{3}\sec^3 x - 47\sec x + C$
19. $\frac{59}{2}\sec x\tan x - \frac{59}{2}\ln|\sec x + \tan x| + C$
20. $\frac{43\sin(3\pi x)}{6\pi} + \frac{43\sin(5\pi x)}{10\pi} + C$
21. $69\csc x + 69\cot x + C$

Trigonometric Substitution

Directions: Evaluate.

22. $\frac{3\sqrt{3}-6+\pi}{648}$
23. $\frac{1}{15}(392 - 28x^2 + 3x^4)\sqrt{x^2+7} + C$
24. $\frac{1}{2}x\sqrt{1-81x^2} + \frac{1}{18}\arcsin(9x) + C$
25. $\frac{1}{2}(x-3)\sqrt{40+6x-x^2} + \frac{49}{2}\arcsin\left(\frac{x-3}{7}\right) + C$
26. $9\sqrt{x^2+x+1} - \frac{9}{2}\ln|1+2x+2\sqrt{x^2+x+1}| + C$
27. $\frac{17x+5}{144\sqrt{15+6x-9x^2}} - \frac{1}{27}\arctan\left(\frac{3x-1}{4}\right) + C$
28. $\frac{1}{2}(x+9)\sqrt{x^2+18x} - \frac{81}{2}\ln|x+9+\sqrt{x^2+18x}| + C$
29. $\frac{7}{4}\arcsin(x^2) + \frac{7}{4}x^2\sqrt{1-x^4} + C$
30. $-\ln(\sqrt{2}-1)$
31. $\frac{x}{\sqrt{9-x^2}} - \arcsin\left(\frac{x}{3}\right) + C$

Integrating Rational Functions

Directions: Evaluate.

32. $\frac{3}{2} - \frac{2}{5}\ln\frac{3}{2}$
33. $2\ln|2x+1| - \ln|x-2| - \frac{1}{x-2} + C$
34. $\frac{1}{2}x^2 - \frac{9}{2}\ln(x^2+9) + \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$
35. $\ln|x-5| - \frac{1}{2}\ln(x^2+9) - \frac{5}{3}\arctan\left(\frac{x}{3}\right) + C$
36. $\arctan x - \frac{5}{2(x^2+1)} + C$
37. $\frac{7}{3}\ln|x-1| - \frac{7}{6}\ln(x^2+x+1) - \frac{7}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C$
38. $\frac{1}{2}\ln\frac{17}{9} - \frac{2}{3}\arctan\frac{5}{3} - \frac{\pi}{6}$
39. $\frac{3}{2}\ln(x^2+6x+10) - \frac{29}{2}\arctan(x+3) - \frac{11x+36}{2(x^2+6x+10)} + C$
40. $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C$
41. $\frac{1}{2}\ln\left|\frac{\sin x}{7\sin x+2}\right| + C$

42. $\frac{1}{97}\ln|e^x-9| - \frac{1}{194}\ln(e^{2x}+16) - \frac{9}{388}\arctan\left(\frac{e^x}{4}\right)+C$

43. $11\left(x-\frac{1}{2}\right)\ln(x^2-x+6) - 22x + 11\sqrt{23}\arctan\left(\frac{2x-1}{\sqrt{23}}\right)+C$

44. $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

45. $\ln|x+4|+C$

46. $\frac{1}{\sqrt{7}}\arctan\left(\frac{x}{\sqrt{7}}\right) - \frac{1}{2(x^2+7)}+C$

47. $4x^2 - 4\ln(x^2+1)+C$

48. $7\ln\frac{3}{2} - \frac{2}{3}$

49. $\frac{1}{6}\ln\frac{2187}{256}$

50. $\ln(3x^2-4x+7)+C$

Integration Strategies

Directions: Evaluate.

51. $\frac{51}{2}\tan^2\theta + 51\ln|\cos\theta|+C$

52. $5e^{\pi/6} - 5e^{-\pi/4}$

53. $\frac{3\sqrt{15}}{5}\arctan\left(\frac{2x^2+1}{\sqrt{15}}\right)+C$

54. $4097/45$

55. $-12x + 6x\ln(x^2-25) + 30\ln\left(\frac{x+5}{x-5}\right)+C$

56. $x - \ln(18+e^x)+C$

57. $\theta\tan\theta - \frac{1}{2}\theta^2 + \ln|\cos\theta|+C$

58. $10\sqrt{9+e^x} - 15x + 30\ln\left(\sqrt{e^x+9}-3\right)+C$

59. $-\frac{4}{3}e^{-x^3}(x^3+1)+C$

60. $\frac{1}{7}\ln\left|\frac{\sqrt{6x+49}-7}{\sqrt{6x+49}+7}\right|+C$

61. $\frac{1}{2}\ln\left(\frac{\sqrt{4x^2+1}-1}{\sqrt{4x^2+1}+1}\right)+C$

62. $e^{-x} + \frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right|+C$

63. $\frac{1}{65}\ln|x-7| - \frac{1}{130}\ln(x^2+16) - \frac{7}{260}\arctan(x/4)+C$

64. $7(\ln x-1)\sqrt{x^2-16} + 28\arctan\frac{\sqrt{x^2-16}}{4}+C$

65. $\frac{7}{15}(x+a)^{15/7} - \frac{7}{8}a(x+a)^{8/7}+C$

66. $24\arctan\sqrt{x}+C$

Improper Integrals

Directions: Evaluate.

67. $2/15$

68. $1/64$

69. $70e^{-2}$

70. $\ln 6$

71. Diverges

72. $7\pi/8$

Limits and L'Hôpital's Rule

Directions: Evaluate.

73. 1

74. 0

75. 1

76. $1/18$

77. 0

78. e

Infinite Sequences

Directions: Determine whether the sequence converges or diverges. If it converges, find the limit.

79. $\lim_{n\to\infty} a_n = 0$

80. $\lim_{n\to\infty} a_n = 1$

81. Diverges

82. $\lim_{n\to\infty} a_n = 0$

83. $\lim_{n\to\infty} a_n = 1$

84. $\lim_{n\to\infty} a_n = 0$

85. Diverges

86. $\lim_{n\to\infty} a_n = 0$

87. $\lim_{n\to\infty} a_n = 1$

88. $\lim_{n\to\infty} a_n = e^{12}$

89. $\lim_{n\to\infty} a_n = \ln 3$

90. Diverges

91. $\lim_{n\to\infty} a_n = 0$

92. Converges to 2

Directions: Determine (a) whether the sequence is increasing, decreasing, or not monotonic; and (b) whether or not the sequence is bounded.

93. (a) not monotonic (b) not bounded

94. (a) decreasing (b) bounded

Infinite Series

Directions: Find the sum of the series.

95. Diverges

96. $5/3$

97. $40/7$

Directions: Determine (a) whether $\{a_n\}$ is convergent, and (b) whether $\sum_{n=1}^{\infty} a_n$ is convergent.

98. (a) yes (b) no

Directions: Determine whether the series converges or diverges. If it is convergent, find the sum.

99. Converges to $38/21$

100. Diverges

101. Diverges

102. Converges to $\dfrac{\cos 1}{1-\cos 1}$

103. Converges to $\dfrac{1+2e}{e-1}$

104. Diverges

105. Converges to $3/2$

106. Converges to $e-1$

Directions: Determine (a) the values of x for which the series converges, and (b) the sum of the series for those values.

107. (a) $(-4, 4)$ (b) $\dfrac{x}{4-x}$

Directions: Answer the questions.

108. No

109. (a) $s_1 = 1/2$, $s_2 = 5/6$, $s_3 = 23/24$, and $s_4 = 119/120$. (b) $s_n = \dfrac{(n+1)!-1}{(n+1)!}$ (c) 1

Integral Test

Directions: Determine whether the series converges or diverges.

110. Converges

111. Converges

112. Converges

113. Converges

114. Converges

Directions: Find the values of p for which the series is convergent.

115. $p > 1$

Comparison Tests

Directions: Determine whether the series converges or diverges.

116. Diverges

117. Converges

118. Converges

119. Converges

120. Diverges

121. Converges

122. Diverges

123. Converges

124. Diverges

Alternating Series Test

Directions: Determine whether the series converges or diverges.

125. Converges

126. Diverges

127. Converges

128. Diverges

129. Converges

130. Converges

131. Converges

Directions: Show that the series is convergent. According to the Alternating Series Sum Estimation Theorem, how many terms of the series do we need to add in order to find the sum to the indicated accuracy?

132. 4 terms

133. 4 terms

134. 5 terms

Directions: Approximate the sum of the series correct to four decimal places.

135. 0.0768

Ratio and Root Tests

Directions: Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

136. Absolutely convergent

137. Divergent

138. Conditionally convergent

139. Absolutely convergent

140. Absolutely convergent

141. Absolutely convergent

142. Absolutely convergent

143. Absolutely convergent

144. Divergent

145. Conditionally convergent

146. Absolutely convergent

147. Divergent

148. Divergent

149. Absolutely convergent

150. Divergent

151. Absolutely convergent

152. Absolutely convergent

153. Divergent

Directions: For each of the following series, is the Ratio Test conclusive or inconclusive?

(a) Inconclusive
(b) Conclusive (convergent)
(c) Conclusive (divergent)
(d) Inconclusive

Strategy for Testing Series

Directions: Test the series for convergence or divergence.

154. Divergent

155. Convergent

156. Divergent

157. Divergent

158. Divergent

159. Divergent

160. Divergent

161. Convergent

162. Convergent

163. Convergent

164. Convergent

Power Series

Directions: Find the radius of convergence and interval of convergence of the series.

165. $R = 1,\ I = [-1, 1)$

166. $R = 1,\ I = (-1, 1]$

167. $R = 1,\ I = [-1, 1]$

168. $R = 1,\ I = (-1, 1)$

169. $R = \infty,\ I = (-\infty, \infty]$

170. $R = 11,\ I = (-11, 11]$

171. $R = 1,\ I = [9, 11]$

172. $R = 1,\ I = [-1, 1]$

173. $R = 7,\ I = (-7, 7)$

Directions: Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?

(a) Convergent

(b) Divergent

(c) Convergent

(d) Divergent

Representations of Functions as Power Series

Directions: Find a power series representation for the function and determine the interval of convergence.

174. $\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{6^{n+1}}$; i.o.c. $(-6, 6)$

175. $\sum_{n=0}^{\infty} \frac{9x^n}{4^{n+1}}$; i.o.c. $(-4, 4)$

176. $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{49^{n+1}}$; i.o.c. $(-7, 7)$

177. $\sum_{n=0}^{\infty}(-1)^n 5^n x^{2n+1}$; i.o.c. $(-1/\sqrt{5}, 1/\sqrt{5})$

Directions: Find a power series representation for the function and determine the radius of convergence.

(a) $\sum_{n=0}^{\infty}(-1)^n (n+1)\frac{x^n}{4^{n+2}};\ R = 4$

(b) $\sum_{n=0}^{\infty}(-1)^n (n+1)(n+2)\frac{x^n}{2^{2n+7}};\ R = 4$

(c) $\sum_{n=2}^{\infty}(-1)^n n(n-1)\frac{x^n}{2^{2n+3}};\ R = 4$

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n};\ R = 1$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n-1};\ R = 1$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^{2n}}{n};\ R = 1$

178. $\ln 2 - \sum_{n=1}^{\infty} \frac{x^n}{n2^n}$; $R = 2$

179. $\sum_{n=3}^{\infty} \frac{n-2}{9^{n-1}} x^n$; $R = 9$

180. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{8^{2n+1}(2n+1)}$; $R = 8$

Directions: Evaluate the indefinite integral as a power series and determine the radius of convergence.

181. $C - \sum_{n=1}^{\infty} \frac{t^n}{n^2}$; $R = 1$

182. $C + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{(2n+1)(2n-1)}$; $R = 1$

183. $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(2n+1)(4n+3)}$; $R = 1$

Directions: Use a power series to approximate the definite integral to six decimal places.

184. 0.001111

185. 0.299969

Directions: Use the formula

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

to compute the indicated value correct to five decimal places.

186. 0.08618

Taylor and Maclaurin Series

Directions: Answer the questions.

187. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{4^n (n+3)}$; $R = 4$

188. $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 5^n x^n}{n}$; $R = 1/5$

189. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{3^{2n+1}(2n+1)!}$; $R = \infty$

190. $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$; $R = \infty$

191. $f(x) = -7+8(x-2)+18(x-2)^2+8(x-2)^3+(x-2)^4$

192. $f(x) = \sum_{n=0}^{\infty} \frac{-6(x+4)^n}{4^{n+1}}$

193. $f(x) = \sum_{n=0}^{\infty} \frac{5(-1)^{n+1}(x-9\pi)^{2n}}{(2n)!}$

194. $f(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!(x-16)^n}{(n!)^2 4^{3n+1}}$

195. 0.83527

196. 0.03490

197. $f(x) = C + \sum_{n=1}^{\infty} \frac{9x^n}{13n \cdot n!}$

198. $f(x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)}$

199. 0.460

200. $125/3$

Curves Defined by Parametric Equations

Directions: Eliminate the parameter to find a Cartesian equation of the curve.

201. $y = (x+6)^2$, $x > -6$

202. $y = e^{x/2}$, $x \geq \ln 36$

Directions: Determine what curve is represented by the parametric equations. Be sure to indicate direction as well as any starting or ending points.

203. It's the circle $x^2 + y^2 = 4$ traced out exactly once in the counterclockwise direction beginning and ending at the point $(2, 0)$.

204. It's the parabola $y = x^2$ with $-1 \le x \le 1$; the point (x, y) moves back and forth infinitely many times along the parabola from $(-1, 1)$ to $(1, 1)$.

Calculus with Parametric Curves

Directions: Answer the questions.

205. $y = x - 1$

206. $y = 2x - 1$

(a) $\dfrac{dy}{dx} = \dfrac{3t+2}{2}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{4t}$

(b) $(0, \infty)$

(a) $(1, -54)$ and $(1, 54)$

(b) $(10, 0)$

(a) $(-3/\sqrt{2}, -1)$, $(-3/\sqrt{2}, 1)$, $(3/\sqrt{2}, -1)$, and $(3/\sqrt{2}, 1)$

(b) $(-3, 0)$ and $(3, 0)$

207. $y = -2x/5$ and $y = 2x/5$

208. $\displaystyle\int_3^5 \sqrt{1+4t^2}\,dt = \frac{1}{4}[10\sqrt{101} + \ln(10 + \sqrt{101}) - 6\sqrt{37} - \ln(6 + \sqrt{37})]$

209. $20\sqrt{10} - 2$

210. $e^3 - e^{-3}$

Polar Coordinates

Directions: Answer the questions.

(a) $(2\sqrt{2}, 7\pi/4)$

(b) $(-2\sqrt{2}, 3\pi/4)$

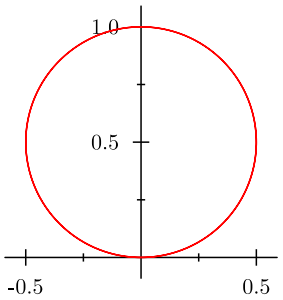
(c) $(2, \pi/3)$

(d) $(-2, 4\pi/3)$

211. $x^2 + (y-3)^2 = 9$; the circle with center $(0, 3)$ and radius 3

Directions: Sketch the graph of the given polar equation.

212.



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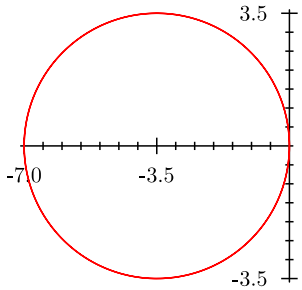

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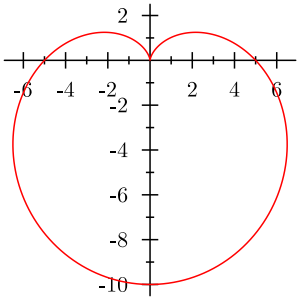

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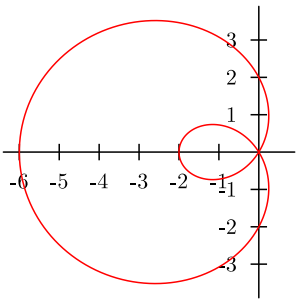

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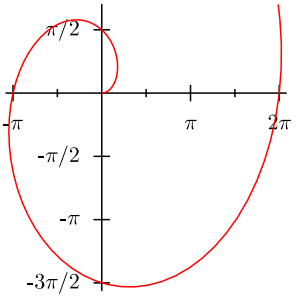

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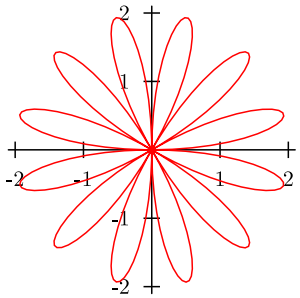

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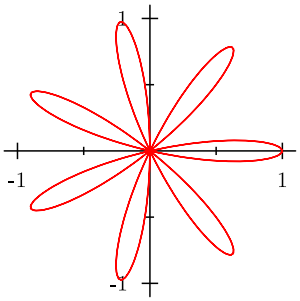

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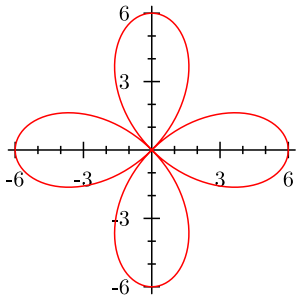

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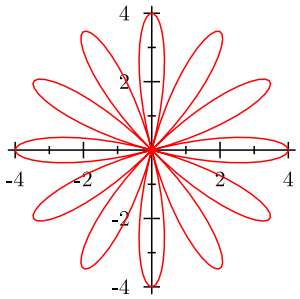

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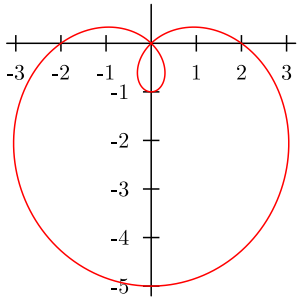

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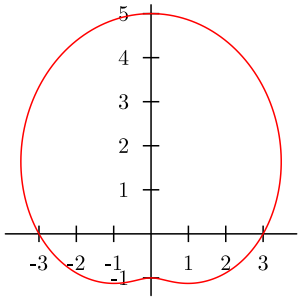

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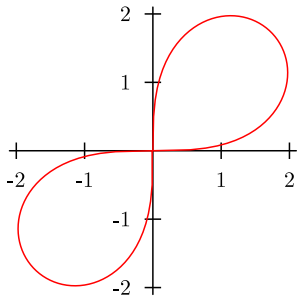

Directions: Answer the questions.

224. $-\sqrt{3}$

(a) $\pi/4,\ 3\pi/4$

(b) $0,\ \pi/2$

(a) $0,\ 2\pi/3,\ 4\pi/3$

(b) $\pi/3,\ \pi,\ 5\pi/3$

Areas in Polar Coordinates

Directions: Answer the questions.

225. $15\pi^2/16$

226. $41\pi/4$

227. $507\pi/2$

228. 11

229. $19\pi/2$

230. 9π

231. $\pi/16$

232. $\dfrac{18\pi - 27\sqrt{3}}{2}$

233. $\dfrac{4\pi + 6\sqrt{3}}{3}$

234. $9\pi + 72$

235. $\dfrac{25(3\sqrt{3} - \pi)}{3}$

236. $8\pi + 6\sqrt{3}$

237. $\dfrac{\pi - 2}{8}$

238. $\dfrac{2 - \sqrt{2}}{2}$

239. $\dfrac{10\pi + 9\sqrt{3}}{12}$

240. $(3/2, \pi/6)$ and $(3/2, 5\pi/6)$

241. $(5, \pi/12)$, $(5, 5\pi/12)$, $(-5, 7\pi/12)$, $(-5, 11\pi/12)$, $(5, 13\pi/12)$, $(5, 17\pi/12)$, $(-5, 19\pi/12)$, and $(-5, 23\pi/12)$

242. π

Areas and Volumes

Directions: Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

243. $49\pi^2/4$

244. $9\pi\left(2\sqrt{2} - \dfrac{5}{2}\right)$

245. 8π

246. $50\pi/9$

247. $\pi/6$

Directions: Answer the questions.

248. $\dfrac{1}{3}\pi h(R^2 + Rr + r^2)$

(a) $436/3$

(b) rectangular solid; $V = b^2h$

(c) square pyramid; $V = \dfrac{1}{3}b^2h$

249. $\dfrac{e^{2\pi} - 13}{4}$

Volumes by Cylindrical Shells

Directions: Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

250. $4\pi/e$

251. $16\pi^3 \ln(4\pi) - 8\pi^3 + \dfrac{\pi}{2}$

252. $64\pi/3$

253. 50π

254. $8\pi/3$

255. $256\pi/15$

256. $27\pi/2$

257. $23\pi/10$

Work

258. 40000 ft-lb

259. 0.96 J

(a) 7.55 J

(b) 120 ft-lb

260. 3.83×10^8 J

261. 5.37×10^5 ft-lb

262. 1058400 J

(a) 750 ft-lb

(b) 562.5 ft-lb

263. 8 cm

264. 16 ft-lb