

1. Evaluate $\int x \sec x \tan x \, dx$

Answer. Integrate by parts, with

$$\begin{aligned} u &= x & dv &= \sec x \tan x \, dx \\ du &= 1 & v &= \sec x \end{aligned}$$

Then we get

$$\begin{aligned} \int x \sec x \tan x \, dx &= x \sec x - \int \sec x \, dx \\ &= x \sec x - \ln |\sec x + \tan x| \end{aligned}$$

□

Answer: $x \sec x - \ln |\sec x + \tan x$

2. True or False?

a) The polynomial $2x^4 - 3x^3 - 19x^2 - 6x + 8$ is divisible by $(x + 1)$ (being divisible means there is 0 remainder after division).

Answer: True

b) The following is a correct first step for breaking up a fraction:

$$\frac{1}{(x-1)^2(x^2+1)(x^2-2x+12)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x^2+1} + \frac{Dx+E}{x^2-2x+12}.$$

Answer: False

3. Evaluate $\int \frac{e^x dx}{e^{2x} - 1}$

Answer. Set $u = e^x$, and so $du = e^x dx$. Then the integral becomes

$$\int \frac{du}{(e^x)^2 - 1} = \int \frac{du}{u^2 - 1} = \int \frac{du}{(u + 1)(u - 1)}$$

Now we use partial fractions to break up this fraction:

$$\frac{1}{(u + 1)(u - 1)} = \frac{A}{u + 1} + \frac{B}{u - 1}$$

$$1 = (u - 1)A + (u + 1)B$$

Plugging in $u = 1, u = -1$ gives $A = -1/2, B = 1/2$.

So now, the problem is

$$-1/2 \int \frac{du}{u + 1} + 1/2 \int \frac{du}{u - 1}$$

which equals

$$-1/2 \ln |u + 1| + 1/2 \ln |u - 1| = 1/2 \ln \left| \frac{u - 1}{u + 1} \right|.$$

Substituting back in for u gives

$$1/2 \ln \left| \frac{e^x - 1}{e^x + 1} \right| = \ln \sqrt{\frac{e^x - 1}{e^x + 1}}.$$

□

Answer: $\ln \sqrt{\frac{e^x - 1}{e^x + 1}}$