1. Solve $\displaystyle\int \frac{5}{(x+3)(x-2)}\,\mathrm{d}x$

Answer. Use partial fractions to break up the integrand.

$$\begin{aligned}\frac{5}{(x+3)(x-2)} &= \frac{A}{x+3} + \frac{B}{x-2}\\ (x+3)(x-2)\left(\frac{5}{(x+3)(x-2)}\right) &= (x+3)(x-2)\left(\frac{A}{x+3} + \frac{B}{x-2}\right)\\ 5 &= (x-2)A + (x+3)B.\end{aligned}$$

Plugging in 2 and -3 for x tells us that $B = 1$ and $A = -1$.
So we have reduced the original problem to

$$\int \frac{-1}{x+3} + \frac{1}{x-2}\,\mathrm{d}x = -\ln(x+3) + \ln(x-2) = \ln\left(\frac{x-2}{x+3}\right).$$

□

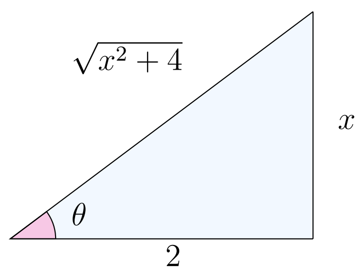
Answer: $\ln\left(\frac{x-2}{x+3}\right) + C$

2. Solve $\int \frac{1}{\sqrt{4+x^2}} \, \mathrm{d}x$

Answer. Use the substitution $x = 2\tan\theta$, $\mathrm{d}x = 2\sec^2\theta \, \mathrm{d}\theta$. Then the problem becomes

$$\int \frac{1}{\sqrt{(4)(1+\tan^2\theta)}} 2\sec^2\theta \, \mathrm{d}\theta = \int \frac{1}{\sec\theta} \sec^2\theta \, \mathrm{d}\theta = \int \sec\theta \, \mathrm{d}\theta = \ln|\sec\theta + \tan\theta|$$

Now, to substitute back in for x we use a triangle, and the identity $\frac{x}{2} = \tan\theta$.



This gives

$$\ln|\sec\theta + \tan\theta| = \ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| = \ln\left|\frac{\sqrt{x^2+4}+x}{2}\right|$$

□

Answer: $\ln\left|\frac{\sqrt{x^2+4}+x}{2}\right|$