1. Find $\int_0^{\pi/2} e^x \cos x \, \mathrm{d}x$

Answer. First find the indefinite integral, and then plug in the values at the end. Integrate by parts, setting

$$\begin{array}{ll} u = \cos x & \mathrm{d}v = e^x \, \mathrm{d}x \\ \mathrm{d}u = -\sin x & v = e^x \end{array}$$

Then we get that

$$\int e^x \cos x \, \mathrm{d}x = e^x \cos x + \int e^x \sin x \, \mathrm{d}x.$$

Now, do the same thing with the integral at the end, and get

$$\int e^x \sin x \, \mathrm{d}x = e^x \sin x - \int e^x \cos x \, \mathrm{d}x.$$

Now just put it all together:

$$\begin{aligned} \int e^x \cos x \, \mathrm{d}x &= e^x \cos x + \left(e^x \sin x - \int e^x \cos x \, \mathrm{d}x \right) \\ \int e^x \cos x \, \mathrm{d}x &= e^x \cos x + e^x \sin x - \underbrace{\int e^x \cos x \, \mathrm{d}x}_{\text{add this to both sides}} \\ \int e^x \cos x \, \mathrm{d}x + \int e^x \cos x \, \mathrm{d}x &= e^x \cos x + e^x \sin x \\ 2 \int e^x \sin x \, \mathrm{d}x &= e^x \sin x + e^x \cos x \\ \int e^x \sin x \, \mathrm{d}x &= \frac{e^x \sin x + e^x \cos x}{2}. \end{aligned}$$

Finally, plug in $\pi/2$ and 0 to get

$$\frac{e^{\pi/2}}{2} - \left(\frac{1}{2} \right).$$

□

Answer: $\frac{e^{\pi/2} - 1}{2}$

2. Find $\int \tan^5 x \sec x \, \mathrm{d}x$

Answer. Use the identities $\frac{\mathrm{d}}{\mathrm{d}x} \sec x = \sec x \tan x$ and $\tan^2 x + 1 = \sec^2 x$. Rewrite the integral as

$$\int \tan^4 x \sec x \tan x \, \mathrm{d}x = \int (\sec^2 - 1)^2 \sec x \tan x \, \mathrm{d}x.$$

Then substitute $u = \sec x, \ \mathrm{d}u = \sec x \tan x \, \mathrm{d}x$. This gives:

$$\begin{aligned} \int (u^2 - 1)^2 \, \mathrm{d}u &= \int (u^4 - 2u^2 + 1) \, \mathrm{d}u \\ &= u^5/5 - 2u^3/3 + u + C \\ &= \frac{\sec x (3 \sec^4 x - 10 \sec^2 x + 15)}{15}. \end{aligned}$$

□

Answer: $\frac{\sec x (3 \sec^4 x - 10 \sec^2 x + 15)}{15} + C$

3. True or False?

a) If $\sec\theta = \frac{5}{3}$, then $\cot\theta = \frac{3}{4}$.

Answer: True

b) $\int e^{x^2} \, \mathrm{d}x = \dfrac{e^{x^2}}{2x}$

Answer: False