

Quiz 3

1. Simplify!

$$\frac{\frac{2x + 5}{x^2 + 2x + 1} - \frac{4}{2x + 2}}{\frac{6x}{2x^3 + 4x^2 + 2x}}$$

Answer. Pick a place to start, and begin simplifying piece by piece. Lets start with the top. Factoring the bottom of the first term gives $(x + 1)^2$, and the bottom of the second term factors to $2(x + 1)$, and so the second term is $\frac{4}{2(x+1)} = \frac{2}{x+1}$.

Now, the top looks like :

$$\frac{2x + 5}{(x + 1)^2} - \frac{2}{x + 1}$$

We need to find a common denominator to combine these fractions. The LCM of the two denominators is $(x + 1)^2$, so we multiply the second one by $\frac{x+1}{x+1}$. This leads to...

$$\begin{aligned} \frac{2x + 5}{x^2 + 2x + 1} - \frac{2}{x + 1} \cdot \frac{x + 1}{x + 1} &= \frac{2x + 5}{(x + 1)^2} - \frac{2(x + 1)}{(x + 1)^2} \\ &= \frac{(2x + 5) - 2(x + 1)}{(x + 1)^2} \\ &= \frac{2x + 5 - 2x - 2}{(x + 1)^2} \\ &= \frac{3}{(x + 1)^2} \end{aligned}$$

So we have simplified the entire top of the fraction to $\frac{3}{(x+1)^2}$.

Now lets work on the bottom. We can factor a $2x$ out of the denominator to get $2x(x^2 + 2x + 1) = 2x(x + 1)^2$. But then this $2x$ cancels with hte $6x$ on top to leave a 3 on top. Therefore the entire bottom of the fraction is $\frac{3}{(x+1)^2}$.

So now that we've simplified the top and the bottom, the expression looks like:

$$\frac{\frac{3}{(x+1)^2}}{\frac{3}{(x+1)^2}}$$

This simplifies to

$$\frac{3}{(x+1)^2} \div \frac{3}{(x+1)^2} = \frac{3}{(x+1)^2} \cdot \frac{(x+1)^2}{3} = 1.$$

□

2. Simplify!

$$\frac{1}{x^2 - x - 6} + \frac{1}{x^2 + x - 2} + \frac{1}{x^2 - 4x + 3}$$

Answer. Step 1: Factor all of the denominators. This gives

$$\frac{1}{(x-3)(x+2)} + \frac{1}{(x+2)(x-1)} + \frac{1}{(x-3)(x-1)}$$

Now, to find the LCM, we look at all of the factors of each denominator. $(x+2)$ is raised to the 1 in the first and second term, but zero in the third, so it is raised to the one in the LCM. Repeating this for the other two, we get that

LCM = $(x-3)(x+2)(x-1)$. Now we make all of the denominators equal by multiplying each term on top and bottom by the appropriate factor.

$$\frac{x-1}{(x-1)(x+2)(x-3)} + \frac{x-3}{(x-1)(x+2)(x-3)} + \frac{x+2}{(x-1)(x+2)(x-3)}$$

Now that the denominators are equal, we just add across the numerators.

$$\frac{(x-1) + (x-3) + (x+2)}{(x-1)(x+2)(x-3)} = \frac{3x-2}{(x-1)(x+2)(x-3)}$$

□

3. True or False?

a) $\frac{1}{x-7} + \frac{1}{7-x} = 0$

b) The polynomial $x^2 - x + 2$ is prime

Answer. a) **True.** Since $7-x = -(x-7)$, the left side becomes

$$\frac{1}{x-7} + \frac{1}{-(x-7)} = \frac{1}{x-7} - \frac{1}{x-7} = 0.$$

b) **True.** There are no two factors of 2 which add up to -1 .

□