Deflatability in Permutation Classes

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(Joint work with Michael Albert, Mike Atkinson, and Jay Pantone)

Plotting Permutations

Definition

If π is a permutation of length n, then the *plot* of π is the set of points

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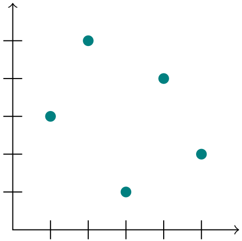

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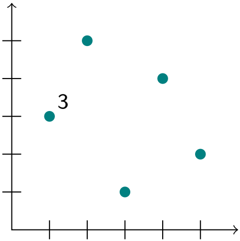

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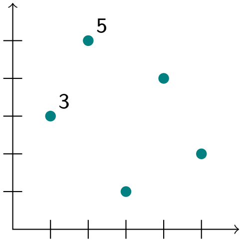

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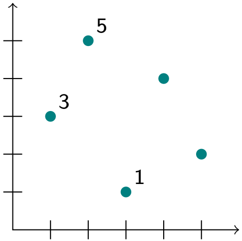

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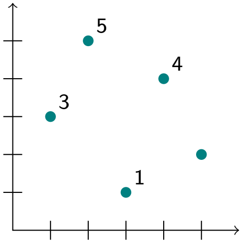

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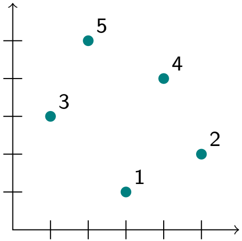

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Dots on a Plane

Definition

Let A and B be two sets of n points in $\mathbb{R}^2$, each with the property that no two points lie on the same horizontal or vertical line.
Say that A is *order isomorphic* to B (denoted $A \sim B$) if A can be transformed into B by stretching, contracting, and translating the axes horizontally and vertically.

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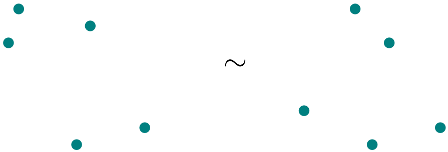

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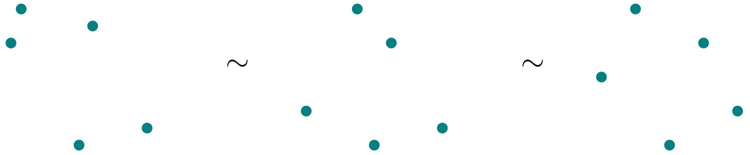

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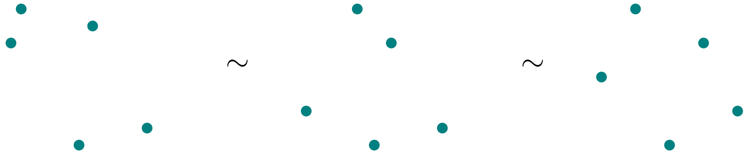

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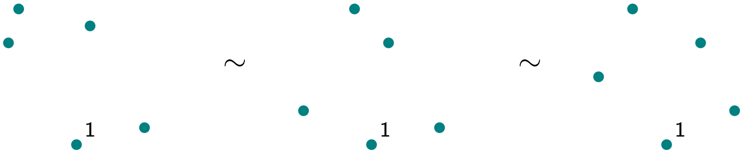

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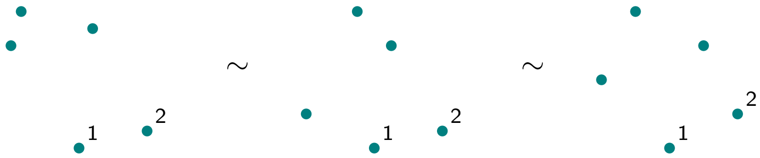

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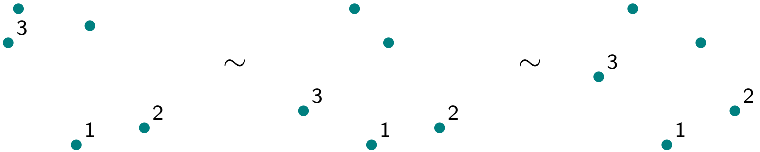

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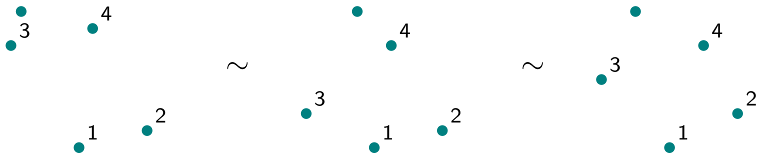

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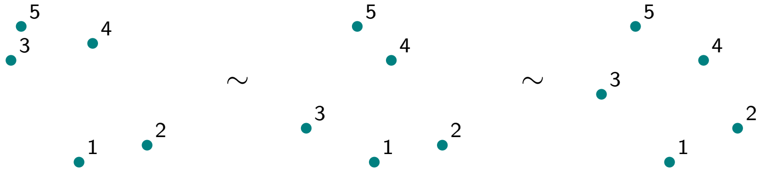

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Permutation Patterns

Definition

Let $\pi = \pi(1)\pi(2)\cdots\pi(n)$ and $\sigma = \sigma(1)\sigma(2)\cdots\sigma(k)$ be two permutations. π *contains* σ *as a pattern* (written $\sigma \prec \pi$) if there is some subsequence $\pi(i_1)\pi(i_2)\ldots\pi(i_k)$ which is order isomorphic to the entries of σ (i.e., $\pi(i_j) < \pi(i_k)$ if and only if $\sigma(j) < \sigma(k)$).

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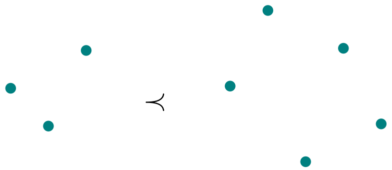

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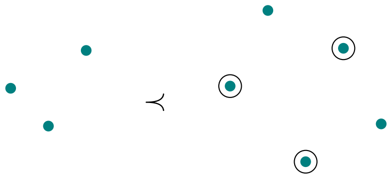

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Simplicity

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An *interval* of a permutation $\pi = \pi_1\pi_2\ldots\pi_n$ is a sequence of consecutive entries $\pi_i\pi_{i+1}\ldots\pi_{i+k}$ whose values form a sequence of consecutive integers. Intervals of size 1 and n are said to be *trivial*.

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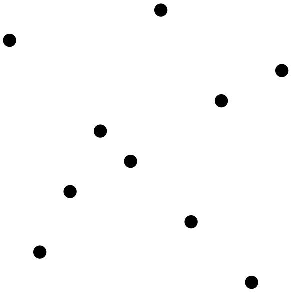

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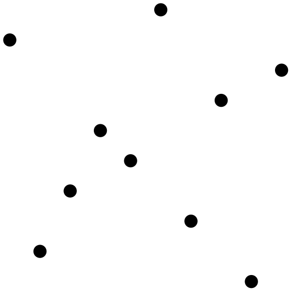

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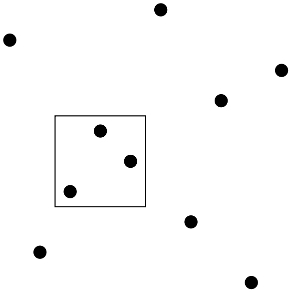

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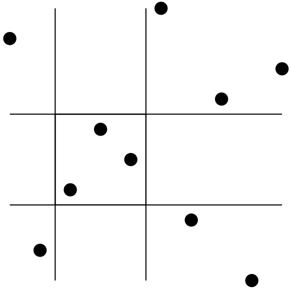

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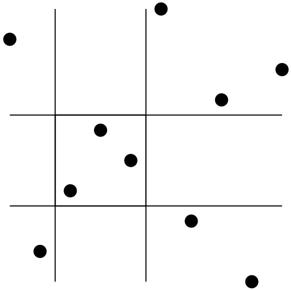

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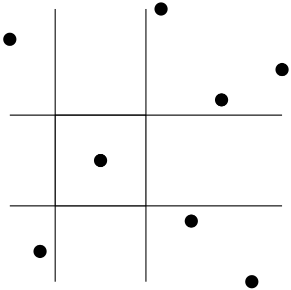

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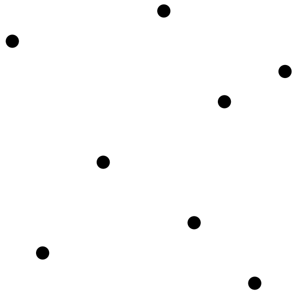

$$\pi = 7\ 2\ 4\ 8\ 3\ 5\ 1\ 6$$

Inflations

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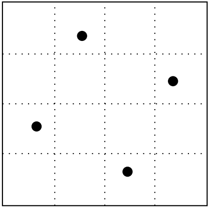

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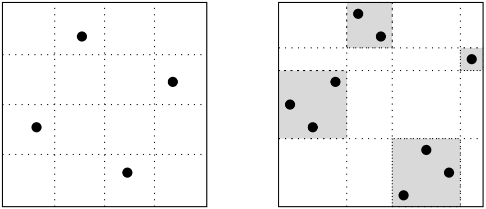

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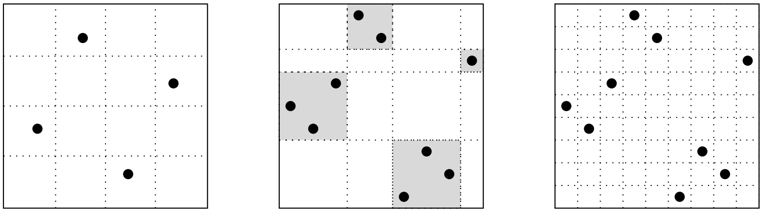

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Substitution Decomposition

Theorem (Albert and Atkinson)

Every permutation π can be written as the inflation of a unique simple permutation σ. Further, if σ has length at least four, then the inflating permutations are uniquely determined.

Class Enumeration

Idea

If we understand the simples within a permutation class, we can use them to understand the class as a whole.

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Example

The only simple permutations in the class $\mathrm{Av}(132)$ are $\{1, 12, 21\}$. Analyzing the ways in which these permutations can be inflated while avoiding 132 leads to a functional equation for the generating function enumerating the class:

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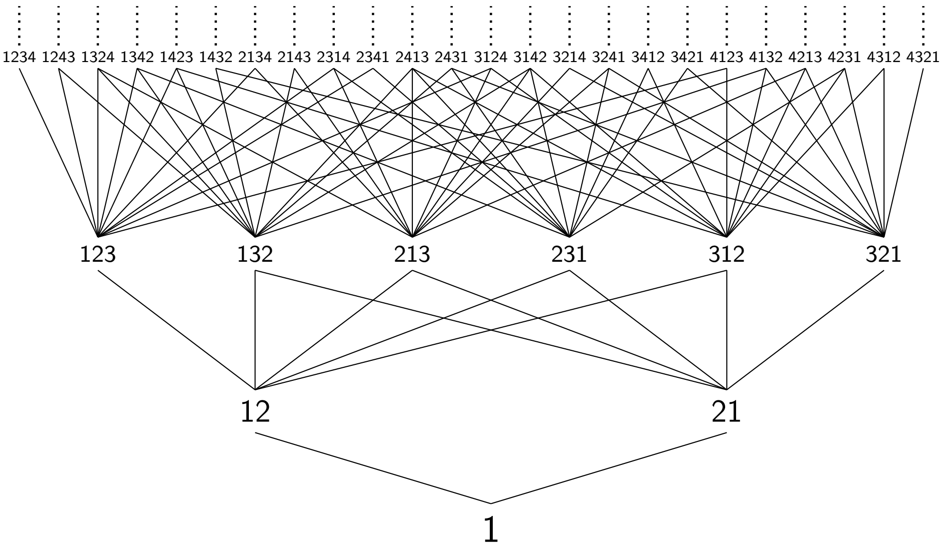
The class Av(123) has infinitely many simple permutations, and cannot be enumerated in this way.

Permutation Classes - Growth Rates



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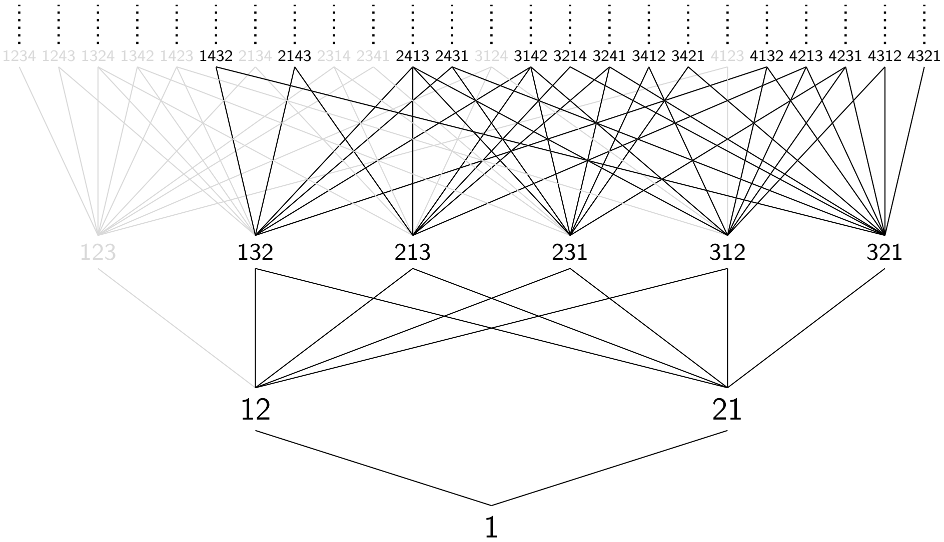

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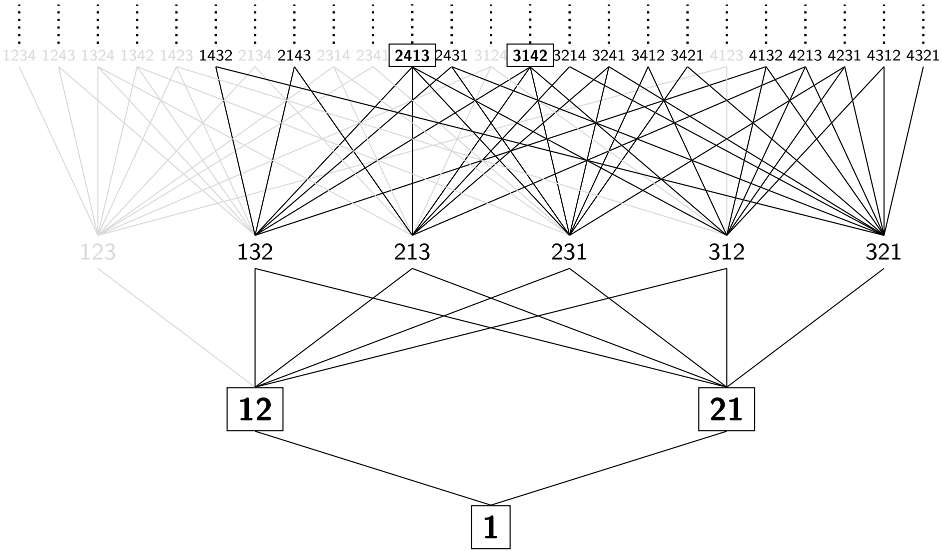

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However...

There is no known instance of a permutation class' simples having positive density.

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A class is *deflatable* if its simple subclass is strictly smaller than the class itself.

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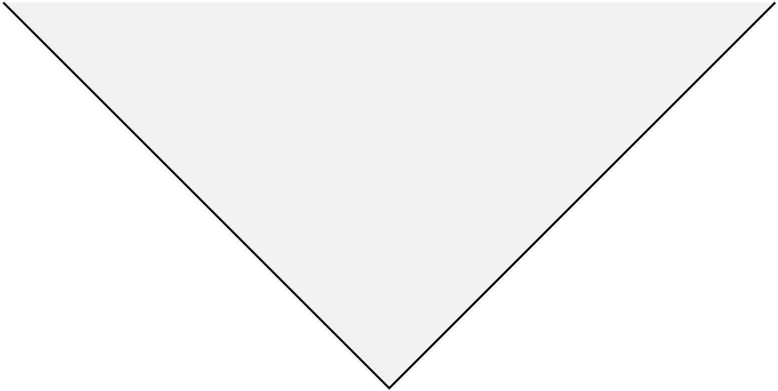

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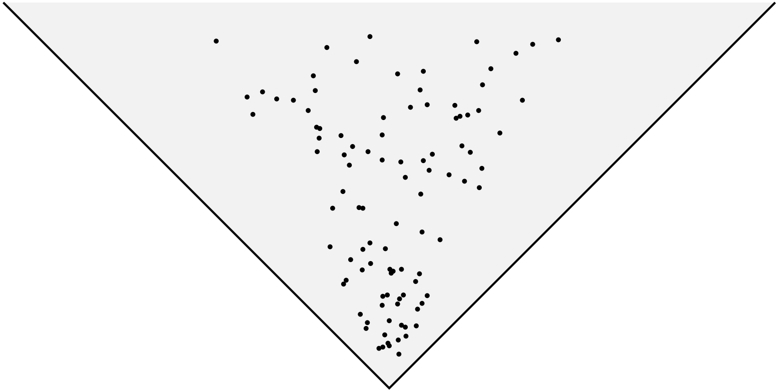

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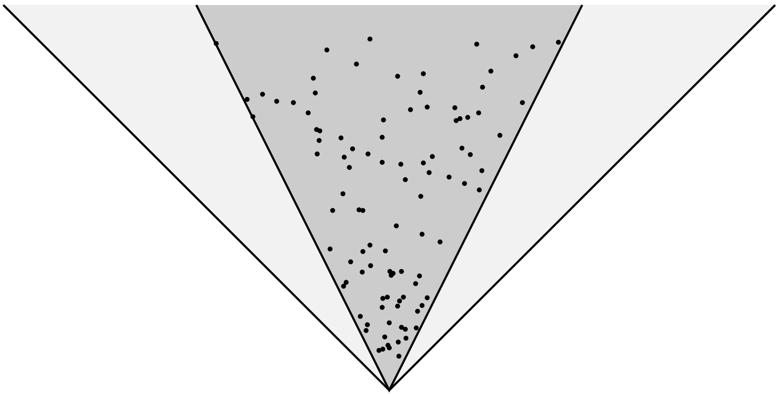

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A permutation class is deflatable if the downward closure of its simples is strictly smaller than the class itself.

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A permutation class C is *not* deflatable if every permutation in the class is contained within a simple permutation in the class.

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Every permutation in $\mathsf{Av}(\pi)$ can be extended to an indecomposable permutation (within the class), except when $\pi \in \{1, 12, 21, 132, 213, 231, 312\}$.

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Lemma

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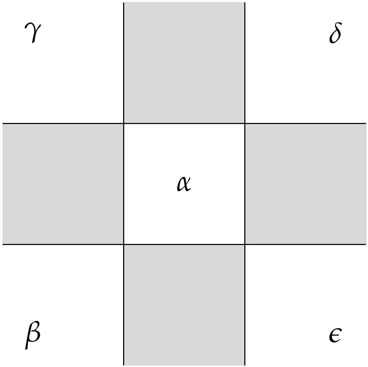
Lemma

A permutation class $\mathsf{Av}(\pi)$ is deflatable if, for every $\omega \in \mathsf{Av}(\pi)$, we can extend ω by a single point which *cuts* a maximal interval of ω.

Proving Non-Deflatability



The Class Av(2413)

Theorem

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Proof

Let $\omega \in \text{Av}(2413)$ be indecomposable and non-simple, and let α be a maximal interval. We need to add a point to ω which cuts α without creating an occurrence of 2413.

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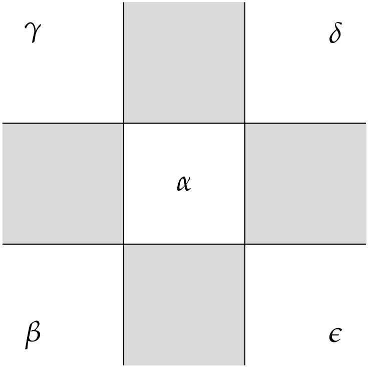

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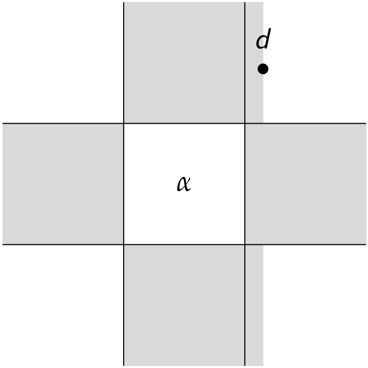

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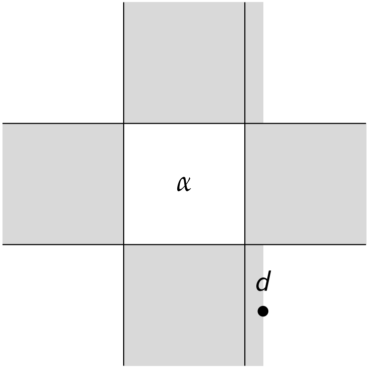

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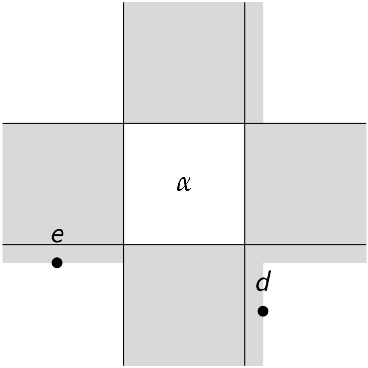

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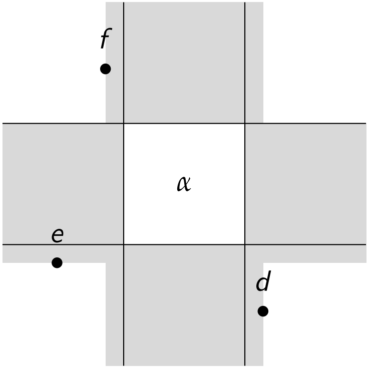

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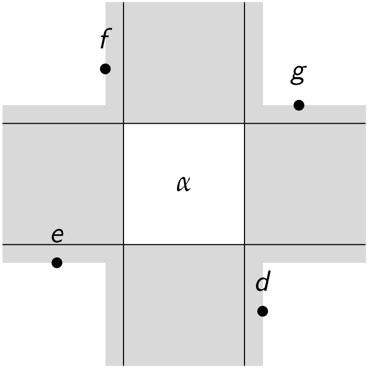

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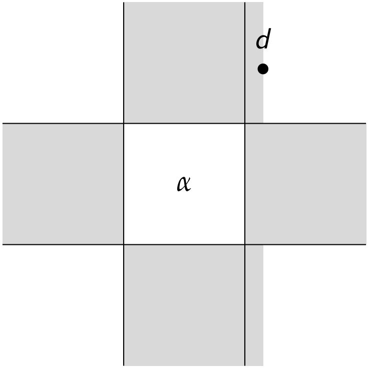

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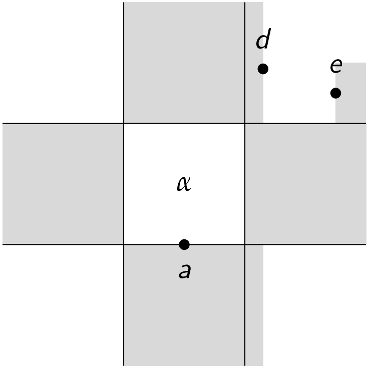

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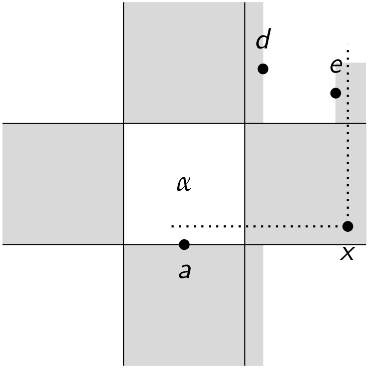

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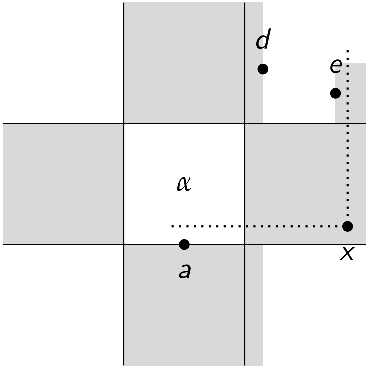

Now claim that this new permutatation avoids 2413

Non-Deflatable Classes

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- $\pi = 1 \oplus \rho$, and ρ starts with a descent, has at least one entry to the right of its smallest which is less than its leftmost, and lacks either an increasing or a decreasing bond

Non-Deflatable Classes

What we've shown

For *most* decomposable patterns π, the class $\mathrm{Av}(\pi)$ is not deflatable.

The only (possible) exceptions all have the form $1 \oplus \rho$, with strict restrictions on ρ.

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Unknown

- Av(146523)
- Av(154623)
- Av(164532)

Deflatable Classes

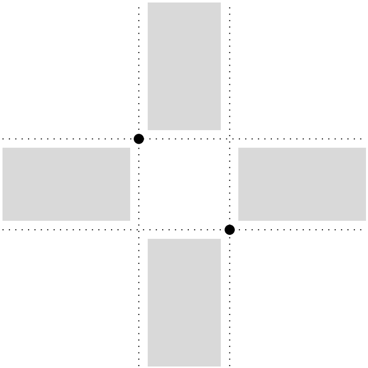
Deflatable Classes

Idea

To prove that a class is deflatable, we need only provide a single *witness*, a permutation in the class which cannot be extended to a simple permutation.

Witnessing Deflation

Let $\omega \in \mathsf{Av}(\pi)$, and consider the permutation diagram of ω.
Denote by gray the *forbidden areas* (those for which inserting an entry creates an occurrence of π).
While adding points to ω, if you ever find yourself in this situation:



Then ω can never be extended to a simple permutation.

A Deflatable Class

Example

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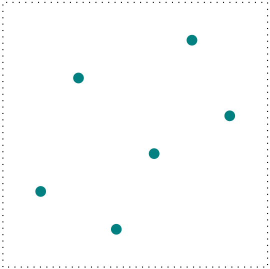
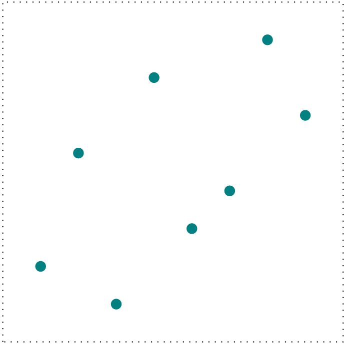


$\pi = 251364$



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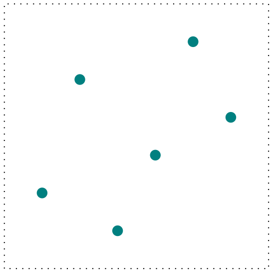

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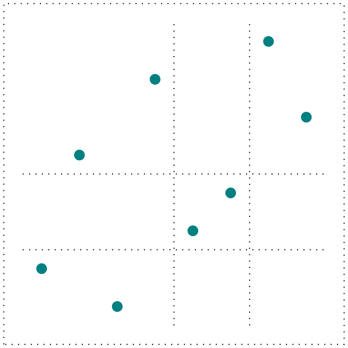

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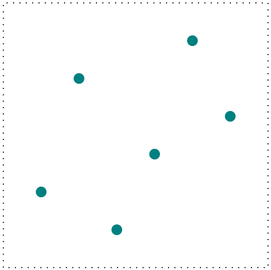

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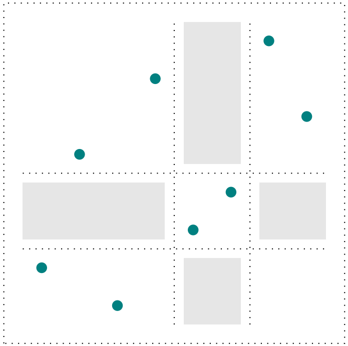

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Infinitely Many Deflatable Classes

Theorem

Let $\pi = 251364$ and θ be any permutation. Then the class $\mathsf{Av}(\pi[1, \theta, 1, 1, 1, 1])$ is deflatable, as evidenced by the witness $\omega = 25173486[1, \theta, 1, \theta, 1, 1, 1, 1]$.

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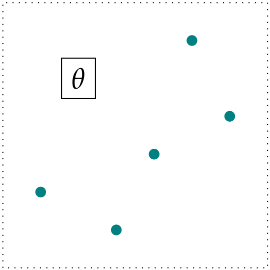

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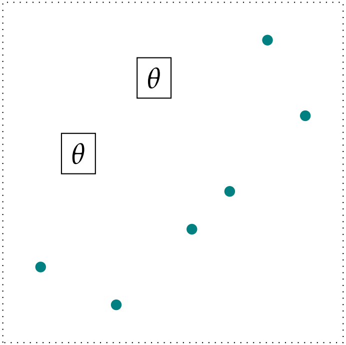

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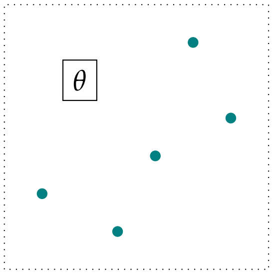

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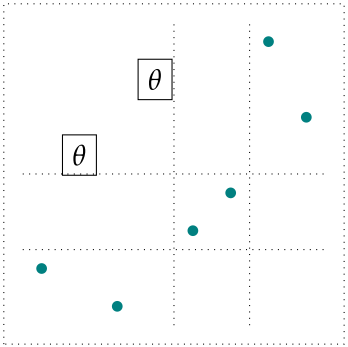

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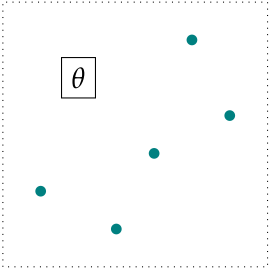

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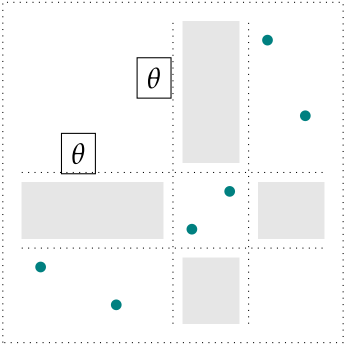

$\omega = 25173486$

More Deflatable Classes

More Deflatable Classes

Class	Witness
Av(134652)	6 8 9 3 4 1 10 14 7 13 5 12 11 2
Av(246135)	4 7 2 9 11 5 6 1 10 3 8
Av(246513)	5 9 3 11 8 2 10 6 7 1 4
Av(251364)	2 5 1 7 3 4 8 6
Av(251463)	2 6 1 8 4 3 7 9 5
Av(254613)	5 9 3 11 2 8 10 6 7 1 4
Av(256413)	4 7 9 2 10 8 5 6 1 3
Av(1523764)	11 18 14 16 8 19 6 7 22 13 1 10 5 24 2 3 9 17 23 4 21 20 15 12
Av(2613475)	2 6 1 3 9 4 5 7 10 8
Av(2631574)	2 6 3 1 9 5 4 8 10 7

What’s Left?

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Are the following classes deflatable?

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- Av(24513)

Conjecture

If π is a *parallel alternation* of length ≥ 6 (i.e., $\pi = 246\ldots(2n)135\ldots(2n-1)$), then the class $\mathrm{Av}(\pi)$ is deflatable.